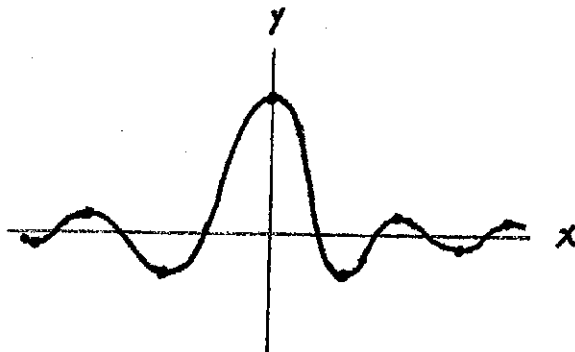


ABSOLUTE MAXIMA AND MINIMA

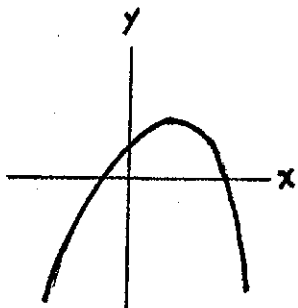


LOTS OF RELATIVE MAXIMA AND MINIMA. ONE "ABSOLUTE MAXIMUM."
TWO "ABSOLUTE MINIMA."

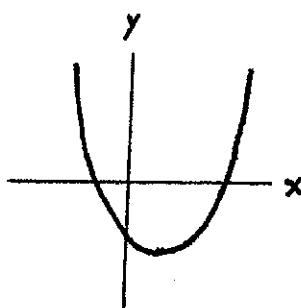
DEFINITION: $f(x)$ HAS AN ABSOLUTE MAXIMUM AT $x = x_0$
IF $f(x) \leq f(x_0)$ FOR EVERY x IN THE DOMAIN OF f .

$f(x)$ HAS AN ABSOLUTE MINIMUM AT $x = x_0$ IF $f(x) \geq f(x_0)$
FOR EVERY x IN THE DOMAIN OF f .

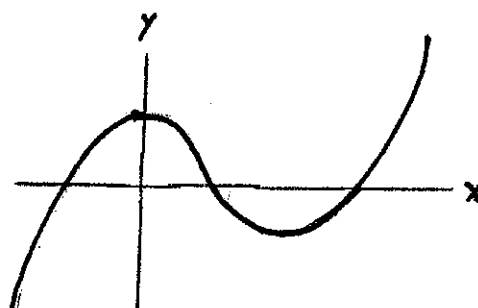
SOME FUNCTIONS HAVE THESE AND SOME DO NOT.



ABSOLUTE MAXIMUM
NO ABSOLUTE
MINIMUM

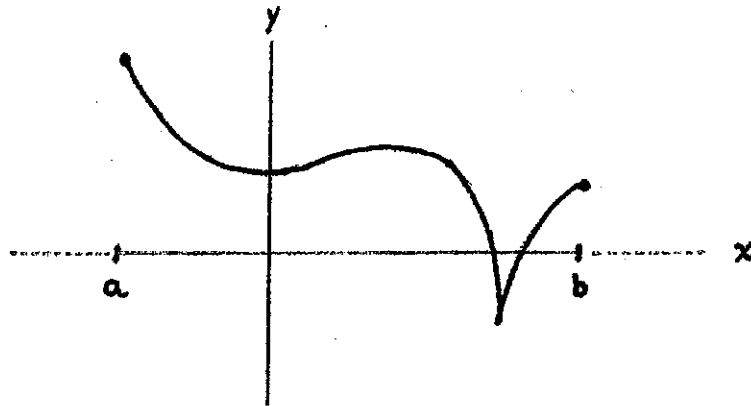


ABSOLUTE MINIMUM
NO ABSOLUTE
MAXIMUM



NEITHER

THERE IS ONE SITUATION IN WHICH BOTH AN ABSOLUTE MAXIMUM AND AN ABSOLUTE MINIMUM MUST EXIST.



THEOREM: IF $f(x)$ IS CONTINUOUS ON A CLOSED INTERVAL $[a, b]$, THEN $f(x)$ HAS AN ABSOLUTE MAXIMUM AND AN ABSOLUTE MINIMUM ON $[a, b]$. THESE CAN ONLY OCCUR IN ONE OF TWO POSSIBLE LOCATIONS:

1. A CRITICAL POINT OF f IN THE OPEN INTERVAL (a, b) .
2. ONE OF THE ENDPOINTS $x = a$ OR $x = b$ (WHICH NEED NOT BE CRITICAL POINTS).

EXAMPLES: FOR EACH OF THE FOLLOWING FIND THE ABSOLUTE MAXIMA AND ABSOLUTE MINIMA, IF THEY EXIST.

1. $f(x) = x^3 - 3x - 1$ ON $[0, 2]$

THE FUNCTION IS A POLYNOMIAL AND THEREFORE CONTINUOUS ON $[0, 2]$.

MUST HAVE BOTH ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM.

POSSIBLE LOCATIONS :

1. CRITICAL POINTS IN $(0, 2)$

$$\begin{aligned} f'(x) &= 3x^2 - 3 = 3(x^2 - 1) \\ &= 3(x-1)(x+1) \end{aligned}$$

CRITICAL POINTS : $x = -1, x = 1$

ONLY $x = 1$ IS IN $(0, 2)$

$$f(1) = 1^3 - 3(1) - 1$$

$$\boxed{f(1) = -3}$$

2. ENDPOINTS :

$$x = 0 : \boxed{f(0) = -1}$$

$$x = 2 : \boxed{f(2) = 1}$$

ABSOLUTE MAXIMUM VALUE OF 1 AT $x = 2$.

ABSOLUTE MINIMUM VALUE OF -3 AT $x = 1$

2. $f(x) = x^3 - 3x - 2$ ON $(-\infty, \infty)$

NOTE THAT

$$\begin{aligned}\lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} (x^3 - 3x - 2) \\ &= \lim_{x \rightarrow \infty} x^3 \\ &= \infty\end{aligned}$$

SO THERE CAN BE NO ABSOLUTE MAXIMUM.

SIMILARLY,

$$\begin{aligned}\lim_{x \rightarrow -\infty} f(x) &= \lim_{x \rightarrow -\infty} (x^3 - 3x - 2) \\ &= \lim_{x \rightarrow -\infty} x^3 \\ &= -\infty\end{aligned}$$

SO THERE CAN BE NO ABSOLUTE MINIMUM.

3. $f(x) = 3x^4 + 4x^3$ ON $(-\infty, \infty)$

THIS TIME

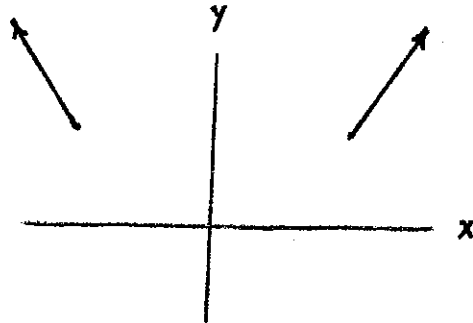
$$\lim_{x \rightarrow \infty} f(x) = \infty$$

AND

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

SO THERE IS NO ABSOLUTE MAXIMUM.

SINCE THE GRAPH LOOKS LIKE



ON THE ENDS, IT ACHIEVES AN ABSOLUTE MINIMUM SOMEWHERE IN BETWEEN (WHERE IT IS CONTINUOUS). THIS MUST OCCUR AT A CRITICAL POINT (WHERE IT TURNS AROUND).

$$\begin{aligned} f'(x) &= 12x^3 + 12x^2 \\ &= 12x^2(x+1) \end{aligned}$$

$$x = 0, -1$$

$$f(0) = 0$$

$$f(-1) = -1$$

ABSOLUTE MINIMUM VALUE OF -1 AT $x = -1$

4. $f(x) = \cos(\sin x)$ ON $[0, \pi]$

CRITICAL POINTS IN $(0, \pi)$: $f'(x) = -\sin(\sin x) \cos x$

$$f'(x) = 0$$

$$-\sin(\sin x) \cos x = 0$$

$$-\sin(\sin x) = 0 \quad \text{OR} \quad \cos x = 0$$