BASIC INTEGRATION TECHNIQUES

IN CHAPTER 8 WE INVESTIGATE MORE SERIOUS "TECHNIQUES OF INTEGRATION" (FINDING ANTIDERIVATIVES) THAN THOSE WE HAVE SEEN SO FAR.

8.1 IS BASICALLY A LIST OF INTEGRATION FORMULAS AND SOME PROBLEMS THAT REVIEW WHAT WE ALREADY KNOW.

BASIC TABLE OF INTEGRALS

1. $\int dx = \int 1\,dx = x + C$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ PROVIDED $n \neq -1$

3. $\int \frac{1}{x} dx = \ln|x| + C$

4. $\int e^x dx = e^x + C$

5. $\int b^x dx = \frac{b^x}{\ln b} + C$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sinh x\,dx = \cosh x + C$

15. $\int \cosh x\,dx = \sinh x + C$

16. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

17. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

18. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$

19. $\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

20. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

21. $\int \frac{1}{1+x^2}\,dx = \arctan x + C$

22. $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$

NOTE: WE ALSO DERIVED SOME ADDITIONAL FORMULAS INVOLVING INVERSE HYPERBOLIC FUNCTIONS, E.G.,

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C$$
$$= \ln(x + \sqrt{x^2+1}) + C.$$

SINCE WE WILL HAVE GENERAL METHODS FOR HANDLING ALL OF THESE SHORTLY, THEY NEED NOT BE MEMORIZED.

NOW, JUST FOR PRACTICE, WE'LL DO SOME EXAMPLES.

1. $\int \sqrt{4+9x}\,dx$

$u = 4+9x$
$du = 9\,dx$

$$\frac{1}{9}\int (4+9x)^{\frac{1}{2}}(9\,dx) = \frac{1}{9}\int u^{\frac{1}{2}}du = \frac{1}{9}\frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{27}(4+9x)^{\frac{3}{2}} + C$$

2. $\int 4x \tan(x^2)\,dx$

$u = x^2$
$du = 2x\,dx$

$$2\int \tan(x^2)(2x\,dx) = 2\int \tan u\,du = 2\ln|\sec u| + C$$
$$= 2\ln|\sec(x^2)| + C$$

3. $\int \frac{1}{4+9x^2}\,dx$

$$\int \frac{1}{4(1+(\frac{3x}{2})^2)}\,dx = \frac{1}{4}\,\frac{2}{3}\int \frac{1}{1+u^2}\,du$$

$u = \frac{3x}{2}$
$du = \frac{3}{2}\,dx$

$$= \frac{1}{6}\arctan u + C$$
$$= \frac{1}{6}\arctan\left(\frac{3x}{2}\right) + C$$

4. $\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$$\int \sec(\ln x)\tan(\ln x)\left(\tfrac{1}{x}dx\right) = \int \sec u \tan u\,du$$
$$= \sec u + C$$
$$= \sec(\ln x) + C$$

5. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{1}{\sqrt{1-(x^2)^2}}(2x\,dx) = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\operatorname{Arcsin} u + C$
$= \frac{1}{2}\operatorname{Arcsin}(x^2) + C$

6. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$

$u = \arctan x$
$du = \frac{1}{1+x^2}\,dx$

$\int e^{\arctan x}\frac{1}{1+x^2}\,dx = \int e^u\,du = e^u + C$
$= e^{\arctan x} + C$

7. $\int (3x+1)\cot(3x^2+2x)\,dx$

$u = 3x^2+2x$
$du = (6x+2)\,dx$
$= 2(3x+1)\,dx$

$\frac{1}{2}\int \cot(3x^2+2x)\,2(3x+1)\,dx = \frac{1}{2}\int \cot u\,du$
$= \frac{1}{2}\ln|\sin u| + C$
$= \frac{1}{2}\ln|\sin(3x^2+2x)| + C$

8. $\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{\ln x}\,\frac{1}{x}\,dx = \int \frac{1}{u}\,du = \ln|u| + c = \ln|\ln x| + c$

9. $\int \sec(\sin\theta)\tan(\sin\theta)\cos\theta\,d\theta$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\int \sec u \tan u\,du = \sec u + C = \sec(\sin\theta) + C$

10. $\int \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \cos(\ln x)\,\frac{1}{x}\,dx = \int \cos u\,du = \sin u + c$

$= \sin(\ln x) + c$

11. $\displaystyle\int \frac{\sinh(x^{-\frac{1}{2}})}{x^{3/2}}\,dx$

$u = x^{-\frac{1}{2}}$

$du = -\frac{1}{2}x^{-3/2}dx$

$$-2\int \sinh(x^{-\frac{1}{2}})(-\tfrac{1}{2}x^{-\frac{3}{2}}dx) = -2\int \sinh u\,du$$
$$= -2\cosh u + C$$
$$= -2\cosh(x^{-\frac{1}{2}}) + C$$

12. $\displaystyle\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

$$\int \frac{1}{\sqrt{4(1-(\frac{e^x}{2})^2)}}\,e^x dx = \frac{1}{2}\int \frac{1}{\sqrt{1-(\frac{e^x}{2})^2}}\,e^x dx$$

$u = \frac{e^x}{2}$

$du = \frac{1}{2}e^x dx$

$$= \frac{1}{2}\cdot 2\int \frac{1}{\sqrt{1-u^2}}\,du$$
$$= \arcsin u + C$$
$$= \arcsin\left(\frac{e^x}{2}\right) + C$$

13. $\displaystyle\int 2^{\pi x}\,dx$

$u = \pi x$

$du = \pi\,dx$

$$\frac{1}{\pi}\int 2^u du = \frac{1}{\pi}\,\frac{2^u}{\ln 2} + C = \frac{2^{\pi x}}{\pi \ln 2} + C$$