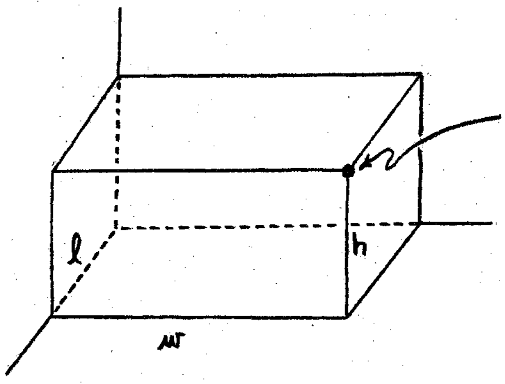
CARTESIAN COORDINATES IN 3-SPACE

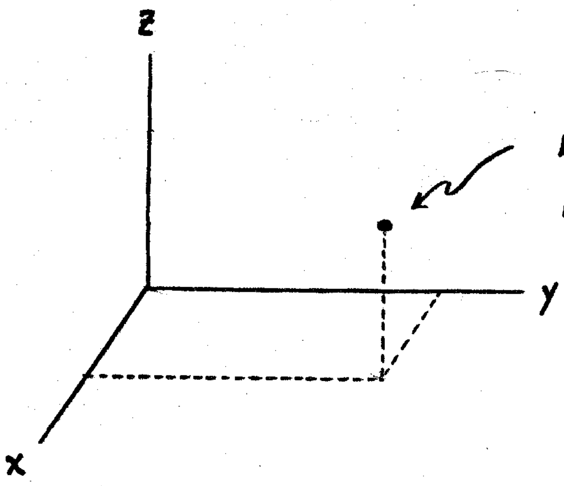
PICTURE A BOX SETTING IN THE CORNER OF THE ROOM :



THE LOCATION OF THE CORNER IS COMPLETELY DETERMINED BY THE THREE NUMBERS

(l, w, h)

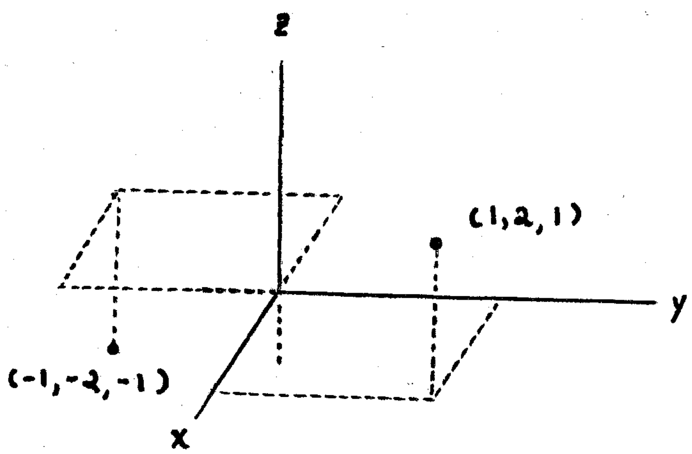
NOW FORGET THE BOX :



EVERY POINT IN THE ROOM IS COMPLETELY DETERMINED BY THREE NUMBERS

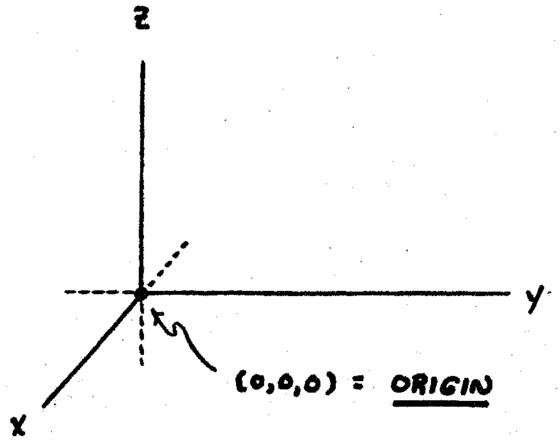
(x, y, z)

ALLOW THE NUMBERS TO BE NEGATIVE AND EVERY POINT IN SPACE IS COMPLETELY DETERMINED BY THREE NUMBERS (x, y, z) :



THESE ARE THE CARTESIAN (RECTANGULAR) COORDINATES OF THE POINTS IN SPACE.

SOME TERMINOLOGY :



COORDINATE AXES :

X-AXIS $(y = z = 0)$

Y-AXIS $(x = z = 0)$

Z-AXIS $(x = y = 0)$

COORDINATE PLANES :

XY-PLANE $(z = 0)$

XZ-PLANE $(y = 0)$

YZ-PLANE $(x = 0)$

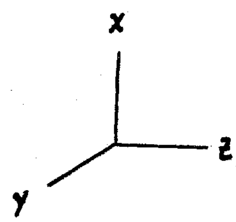
THE COORDINATE PLANES DIVIDE SPACE UP INTO EIGHT REGIONS, CALLED OCTANTS.

1ST OCTANT : $x \geq 0$, $y \geq 0$ AND $z \geq 0$

(" THE ROOM ")

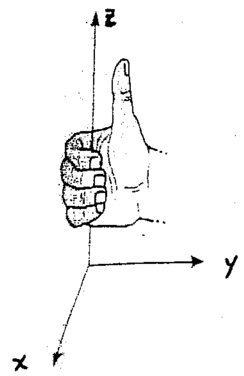
NUMBER THE OTHERS ANY WAY YOU LIKE. NO ONE CARES.

THE AXES CAN BE LABELLED IN OTHER WAYS (E.G.,



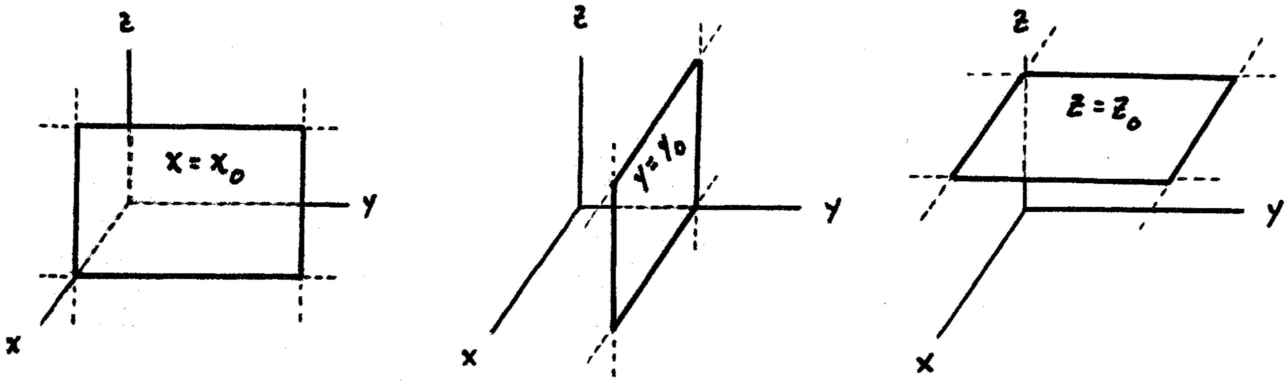
)

BUT WE WILL ALWAYS DO IT IN SUCH A WAY THAT THE COORDINATE SYSTEM IS RIGHT-HANDED :

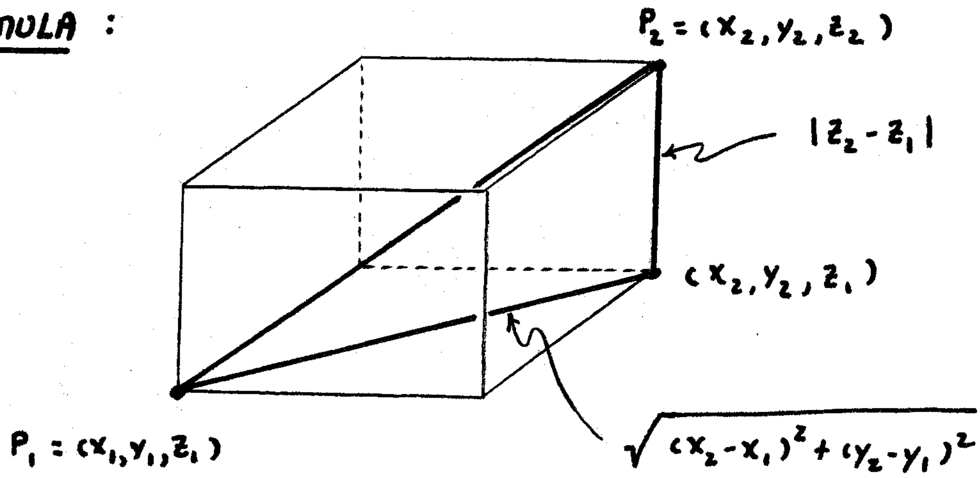


CURL THE FINGERS OF YOUR RIGHT HAND FROM THE POSITIVE X-AXIS TO THE POSITIVE Y-AXIS. YOUR THUMB SHOULD BE POINTING IN THE DIRECTION OF THE POSITIVE Z-AXIS.

PLANES OF CONSTANT x, y AND z :



DISTANCE FORMULA :



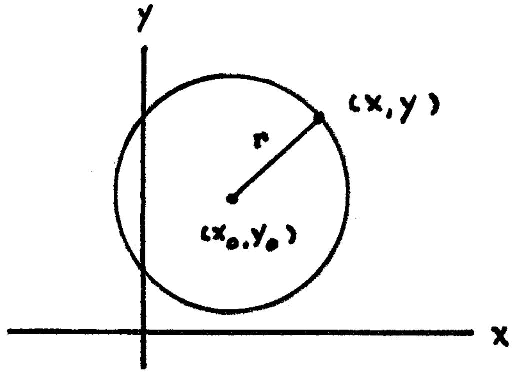
DISTANCE BETWEEN P_1 AND P_2 IS, BY THE PYTHAGOREAN THEOREM,

$$\sqrt{\left(\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right)^2 + |z_2-z_1|^2}$$

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

JUST AS THE SET OF ALL POINTS (x,y) IN THE PLANE SATISFYING

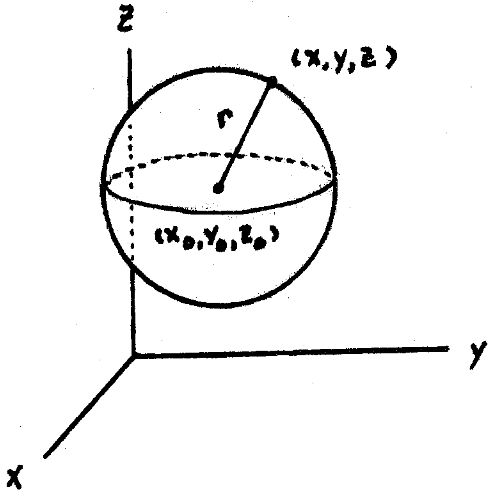
$(x-x_0)^2 + (y-y_0)^2 = r^2$ IS THE CIRCLE OF RADIUS r ABOUT (x_0,y_0)



SO THE SET OF ALL POINTS (x,y,z) IN SPACE SATISFYING

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

IS THE <u>SPHERE</u> OF RADIUS r ABOUT (x_0,y_0,z_0).



<u>EXAMPLES</u> :

1. $(x-1)^2 + (y-\frac{1}{2})^2 + (z-2)^2 = \frac{1}{4}$

   CENTER : $(1, \frac{1}{2}, 2)$ RADIUS : $\frac{1}{2}$

2. $(x-1)^2 + y^2 + (z+4)^2 = 5$

$(x-1)^2 + (y-0)^2 + (z-(-4))^2 = (\sqrt{5})^2$

CENTER : $(1,0,-4)$ RADIUS : $\sqrt{5}$

3. $x^2+y^2+z^2 = 1$

$(x-0)^2 + (y-0)^2 + (z-0)^2 = 1^2$

CENTER : $(0,0,0)$ RADIUS : 1

THIS IS CALLED <u>THE</u> <u>UNIT</u> <u>SPHERE</u>.

4. $x^2 + y^2 + z^2 + 8z + 4 = 0$

$(x-0)^2 + (y-0)^2 + (z^2+8z \qquad) = -4$

COMPLETE THE SQUARE

$(x-0)^2 + (y-0)^2 + (z^2+8z+16) = -4+16$

$(x-0)^2 + (y-0)^2 + (z+4)^2 = 12$

CENTER : $(0,0,-4)$ RADIUS : $\sqrt{12} = 2\sqrt{3}$

5. $x^2+y^2+z^2-2x-4y+8z+17 = 0$

$(x^2-2x \qquad) + (y^2-4y \qquad) + (z^2+8z \qquad) = -17$

$(x^2-2x+1) + (y^2-4y+4) + (z^2+8z+16) = -17+1+4+16$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 4$

CENTER : $(1,2,-4)$ RADIUS : 2

NOTICE THAT

$$x^2+y^2+z^2-2x-4y+8z+21=0$$

WOULD GIVE

$$(x-1)^2+(y-2)^2+(z+4)^2=0$$

SO THE GRAPH IS JUST ONE POINT

$$(x,y,z)=(1,2,-4)$$

AND

$$x^2+y^2+z^2-2x-4y+8z+22=0$$

WOULD GIVE

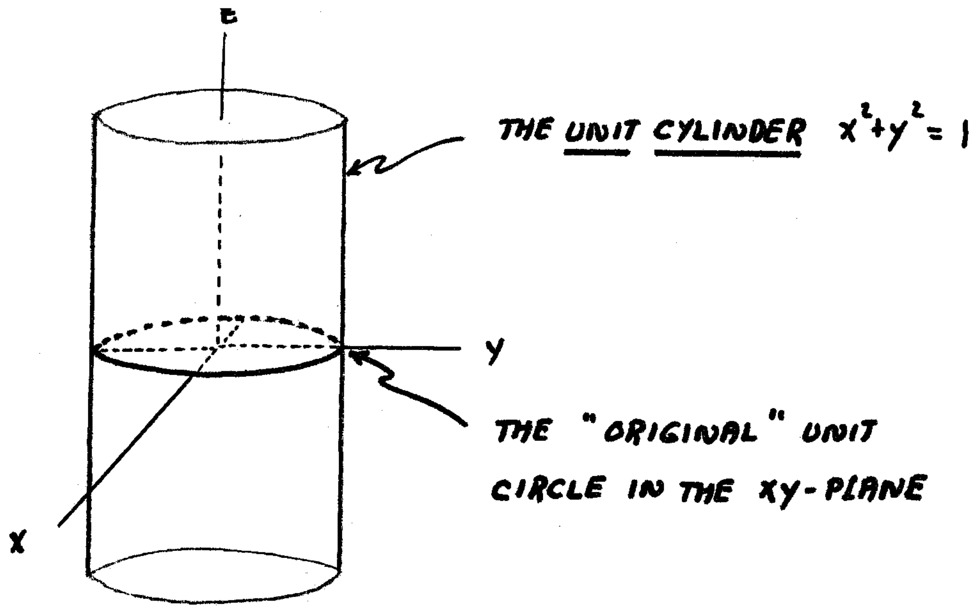
$$(x-1)^2+(y-2)^2+(z+4)^2=-1$$

WHICH HAS NO SOLUTIONS (THE GRAPH IS EMPTY).

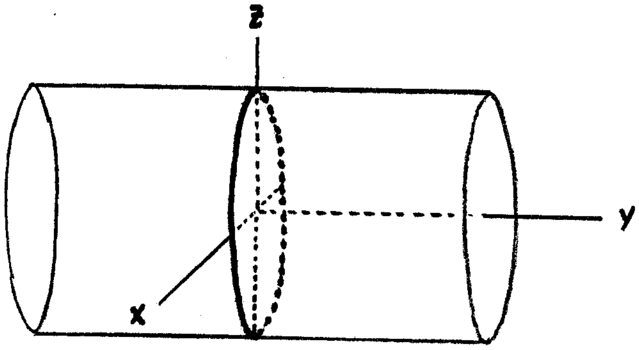
A FEW MORE SIMPLE GRAPHS:

THE GRAPH OF $x^2+y^2=1$ IN THE XY-PLANE IS THE UNIT CIRCLE (ALL POINTS (x,y) IN THE PLANE THAT SATISFY THE EQUATION).

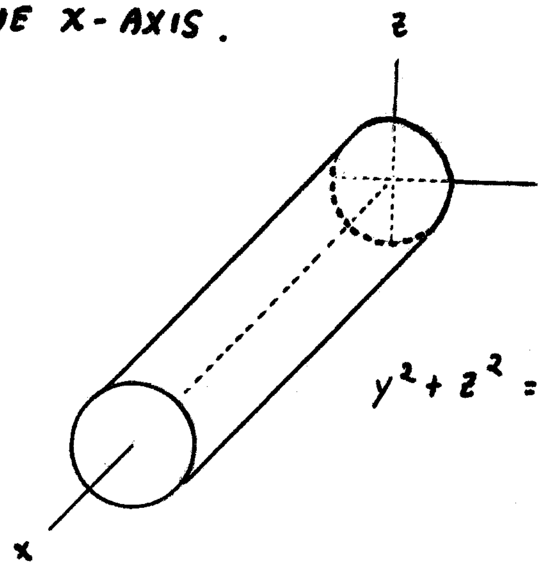
ONE COULD ALSO ASK FOR THE GRAPH OF THE SAME EQUATION $x^2+y^2=1$ IN SPACE (ALL POINTS (x,y,z) IN SPACE THAT SATISFY THE EQUATION) AND THIS IS MUCH LARGER (ANY POINT SETTING ABOVE, BELOW, OR ON THE UNIT CIRCLE IN THE XY-P ANE):



SIMILARLY, THE GRAPH OF $x^2+z^2=1$ IN SPACE IS A CYLINDER OF RADIUS 1 ALONG THE y-AXIS AND THE GRAPH OF $y^2+z^2=1$ IS A CYLINDER OF RADIUS 1 ALONG THE X-AXIS.



$x^2+z^2=1$

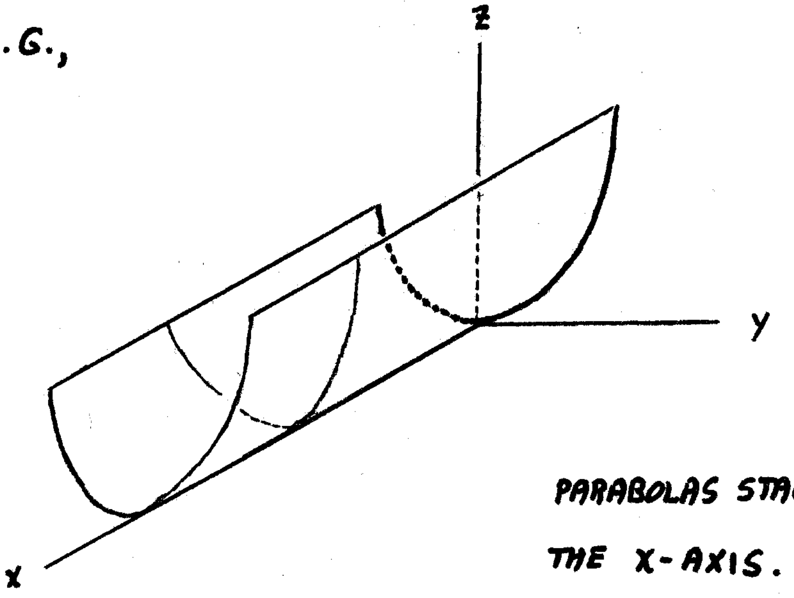


$y^2+z^2=1$

TO GRAPH, IN SPACE, AN EQUATION WITH ONLY 2 OF THE 3 VARIABLES PRESENT, GRAPH IT FIRST IN THE PLANE OF THE VARIABLES THAT ARE PRESENT AND THEN "PULL" IT IN THE DIRECTION OF THE MISSING VARIABLE.

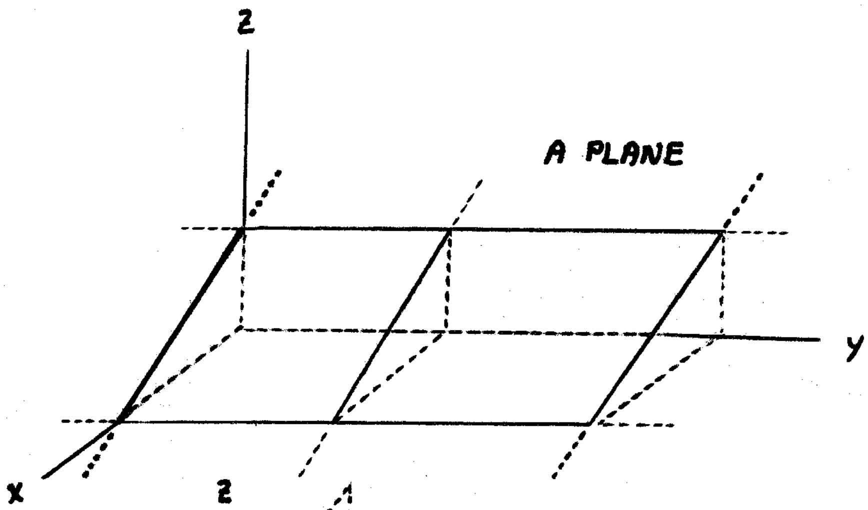
UNFORTUNATELY, ANY SURFACE OBTAINED IN THIS WAY IS CALLED A <u>CYLINDER</u>, E.G.,

1. $z = y^2$



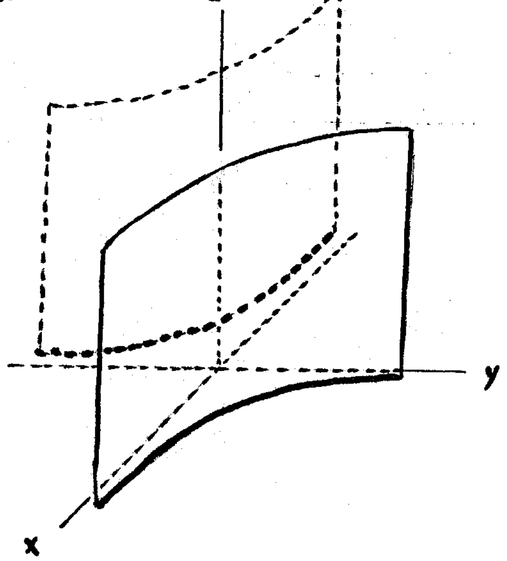
PARABOLAS STACKED ALONG THE X-AXIS.

2. $2x + 3z = 6$



A PLANE

3. $xy = 1$



HYPERBOLAS STACKED IN THE Z-DIRECTION