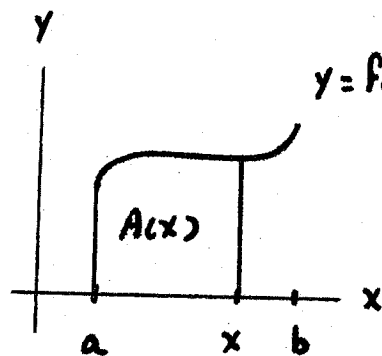


INDEFINITE INTEGRALS

LAST TIME WE DESCRIBED A LONG AND ELABORATE PROCEDURE FOR COMPUTING AREAS (AND NET SIGNED AREAS) FOR A FUNCTION $f(x)$ OVER AN INTERVAL $[a, b]$.

WE WILL RETURN TO THIS PROCEDURE A BIT LATER.

NOW I WANT TO REMIND YOU WHY WE CARE ABOUT SUCH AREAS:



$$A'(x) = f(x)$$

THIS IS THE MOTIVATION BEHIND THE FOLLOWING DEFINITION:

GIVEN A FUNCTION $f(x)$ ON SOME INTERVAL I

AN ANTIDERIVATIVE FOR $f(x)$ ON I IS ANOTHER

FUNCTION $F(x)$ SATISFYING

$$F'(x) = f(x)$$

FOR ALL x IN I .

E.G., IF $f(x) = 3x^2$, THEN ALL OF THE FOLLOWING ARE ANTIDERIVATIVES FOR $f(x)$ ON $(-\infty, \infty)$:

$$x^3, \quad x^3 + 5, \quad x^3 - \frac{\pi^2}{6}, \dots$$

$$x^3 + C \quad \text{FOR ANY CONSTANT } C$$

NOTE: LAST TERM WE PROVED, FROM THE MEAN VALUE THEOREM, THAT $x^3 + C$ DESCRIBES ALL OF THE ANTIDERIVATIVES OF $3x^2$ ON $(-\infty, \infty)$.

$$f(x) \xrightarrow{\text{DIFFERENTIATION}} f'(x)$$

$$f(x) \xrightarrow[\text{INTEGRATION}]{\text{ANTIDIFFERENTIATION}} F(x) \text{ FOR WHICH } F'(x) = f(x)$$

INTEGRAL NOTATION :

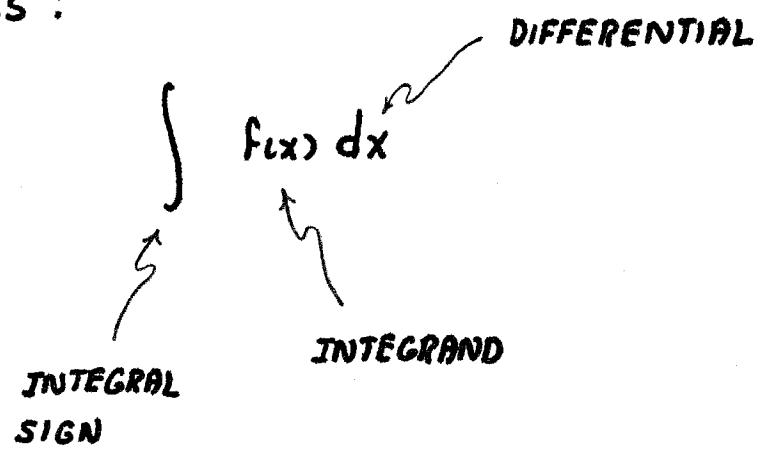
$$\int f(x) dx = \text{COLLECTION OF ALL ANTIDERIVATIVES FOR } f(x) \text{ ON } I \\ = F(x) + C$$

WHERE $F(x)$ IS ANY FUNCTION SATISFYING $F'(x) = f(x)$ ON I AND C IS AN ARBITRARY CONSTANT

E.G.,

$$\int 3x^2 dx = x^3 + C$$

THE SYMBOLS :



$$\int f(x) dx = \underline{\text{INDEFINITE INTEGRAL OF } f(x)}$$

A FEW EXAMPLES :

$$\int \cos x dx = \sin x + C \quad \left(\frac{d}{dx} (\sin x) = \cos x \right)$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C \quad \left(\frac{d}{dx} (\arcsin x) = \frac{1}{\sqrt{1-x^2}} \right)$$

$$\int x^5 dx = \frac{1}{6} x^6 + C \quad \left(\frac{d}{dx} \left(\frac{1}{6} x^6 \right) = x^5 \right)$$

$$\int t^5 dt = \frac{1}{6} t^6 + C \quad \left(\frac{d}{dt} \left(\frac{1}{6} t^6 \right) = t^5 \right)$$

BASIC TABLE OF INTEGRALS

$$1. \int \cos x \, dx = \sin x + C$$

$$2. \int \sin x \, dx = -\cos x + C$$

$$3. \int \sec^2 x \, dx = \tan x + C$$

$$4. \int \csc^2 x \, dx = -\cot x + C$$

$$5. \int \sec x \tan x \, dx = \sec x + C$$

$$6. \int \csc x \cot x \, dx = -\csc x + C$$

$$7. \int e^x \, dx = e^x + C$$

$$8. \int b^x \, dx = \frac{b^x}{\ln b} + C$$

$$9. \int 1 \, dx = \int dx = x + C$$

$$10. \int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

PROVIDED $n \neq -1$

$$11. \int \frac{1}{x} \, dx = \ln |x| + C$$

$$12. \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$

$$13. \int \frac{1}{1+x^2} \, dx = \arctan x + C$$

$$14. \int \frac{1}{x\sqrt{x^2-1}} \, dx = \operatorname{arcsec} |x| + C$$

AND A FEW SIMPLE RESULTS ON PUTTING THEM TOGETHER: SINCE

$(f(x) \pm g(x))' = f'(x) \pm g'(x)$ AND $(c f(x))' = c f'(x)$,

$$15. \int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$16. \int c f(x) \, dx = c \int f(x) \, dx$$

EXAMPLES :

$$1. \int x^5 dx = \frac{x^{5+1}}{5+1} + C = \frac{1}{6} x^6 + C$$

$$2. \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} x^{\frac{3}{2}} + C$$

$$\begin{aligned} 3. \int (x^2 + x) dx &= \int x^2 dx + \int x dx \\ &= \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + C \\ &= \frac{1}{3} x^3 + \frac{1}{2} x^2 + C \end{aligned}$$

NOTE: ONE ARBITRARY
CONSTANT WILL SUFFICE SINCE
THE SUM OF TWO IS JUST
ANOTHER ARBITRARY CONSTANT

$$\begin{aligned} 4. \int \left(\frac{1}{\sqrt{u}} - 4 \sec^2 u \right) du &= \int (u^{-\frac{1}{2}} - 4 \sec^2 u) du = \\ \int u^{-\frac{1}{2}} du - 4 \int \sec^2 u du &= \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 4 \tan u + C = \\ 2u^{\frac{1}{2}} - 4 \tan u + C &= 2\sqrt{u} - 4 \tan u + C \end{aligned}$$

$$5. \int \frac{5}{\sqrt{1-t^2}} dt = 5 \int \frac{1}{\sqrt{1-t^2}} dt = 5 \text{ ARCSIN } t + C$$

$$\begin{aligned}
6. \quad \int \frac{x^3 + 3x - 1}{x^2} dx &= \int \left(\frac{x^3}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2} \right) dx = \\
&= \int \left(x + 3\left(\frac{1}{x}\right) - x^{-2} \right) dx = \int x dx + 3 \int \frac{1}{x} dx - \int x^{-2} dx = \\
&= \frac{x^{1+1}}{1+1} + 3 \ln|x| - \frac{x^{-2+1}}{-2+1} + C = \frac{1}{2}x^2 + 3 \ln|x| - \frac{x^{-1}}{-1} + C = \\
&= \frac{1}{2}x^2 + 3 \ln|x| + \frac{1}{x} + C
\end{aligned}$$

7. SOLVE THE INITIAL VALUE PROBLEM

$$\begin{cases} \frac{dy}{dx} = \sec^2 x - \sin x \\ y\left(\frac{\pi}{4}\right) = 1 \end{cases}$$

ALL THIS ASKS FOR IS AN ANTIDERIVATIVE y FOR $\sec^2 x - \sin x$ WHOSE VALUE AT $\frac{\pi}{4}$ IS 1.

ALL ANTIDERIVATIVES FOR $\sec^2 x - \sin x$ ARE

$$\begin{aligned}
\int (\sec^2 x - \sin x) dx &= \int \sec^2 x dx - \int \sin x dx = \\
&= \tan x - (-\cos x) + C = \tan x + \cos x + C
\end{aligned}$$

THUS, FOR SOME CHOICE OF C ,

$$y(x) = \tan x + \cos x + C$$

AT $x = \frac{\pi}{4}$,

$$1 = y\left(\frac{\pi}{4}\right) = \tan \frac{\pi}{4} + \cos \frac{\pi}{4} + C = 1 + \frac{\sqrt{2}}{2} + C$$

SO

$$C = -\frac{\sqrt{2}}{2}$$

AND

$$y(x) = \tan x + \cos x - \frac{\sqrt{2}}{2}.$$