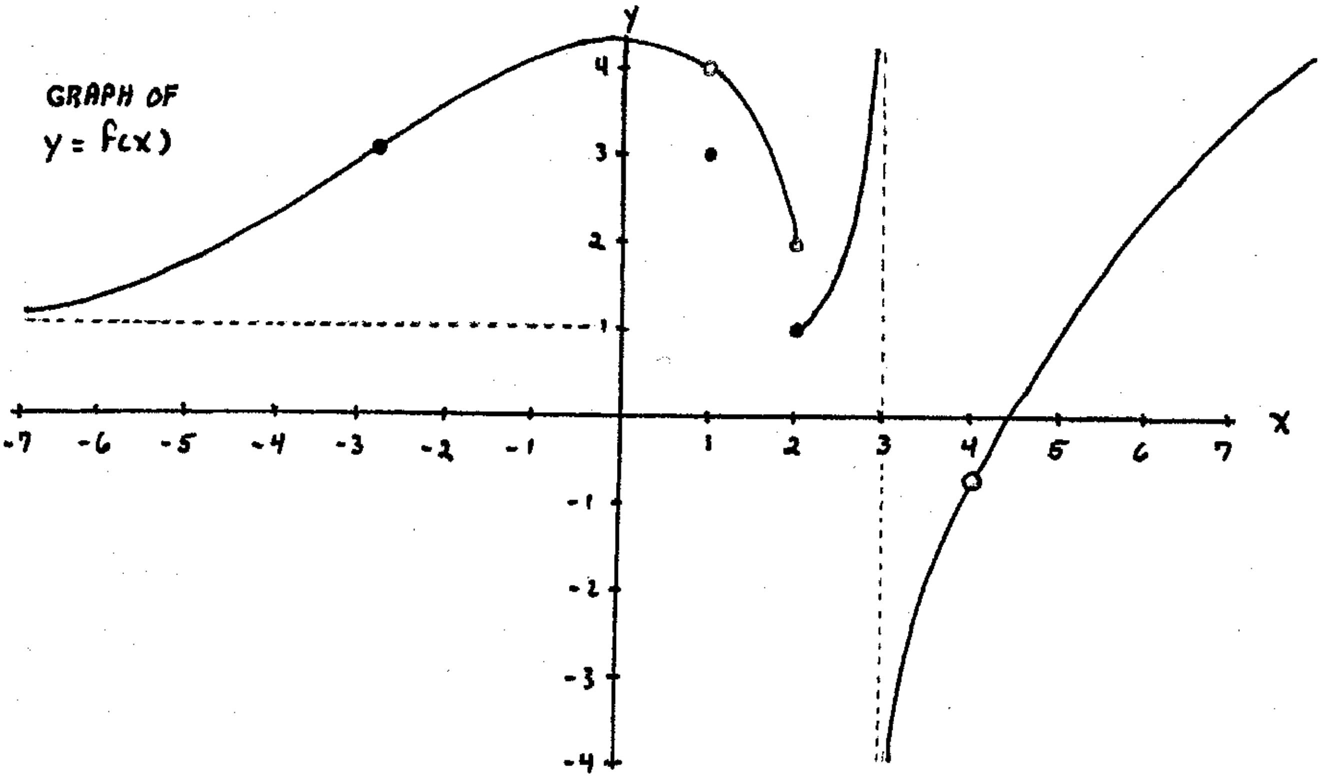
LIMITS : INTUITIVE APPROACH

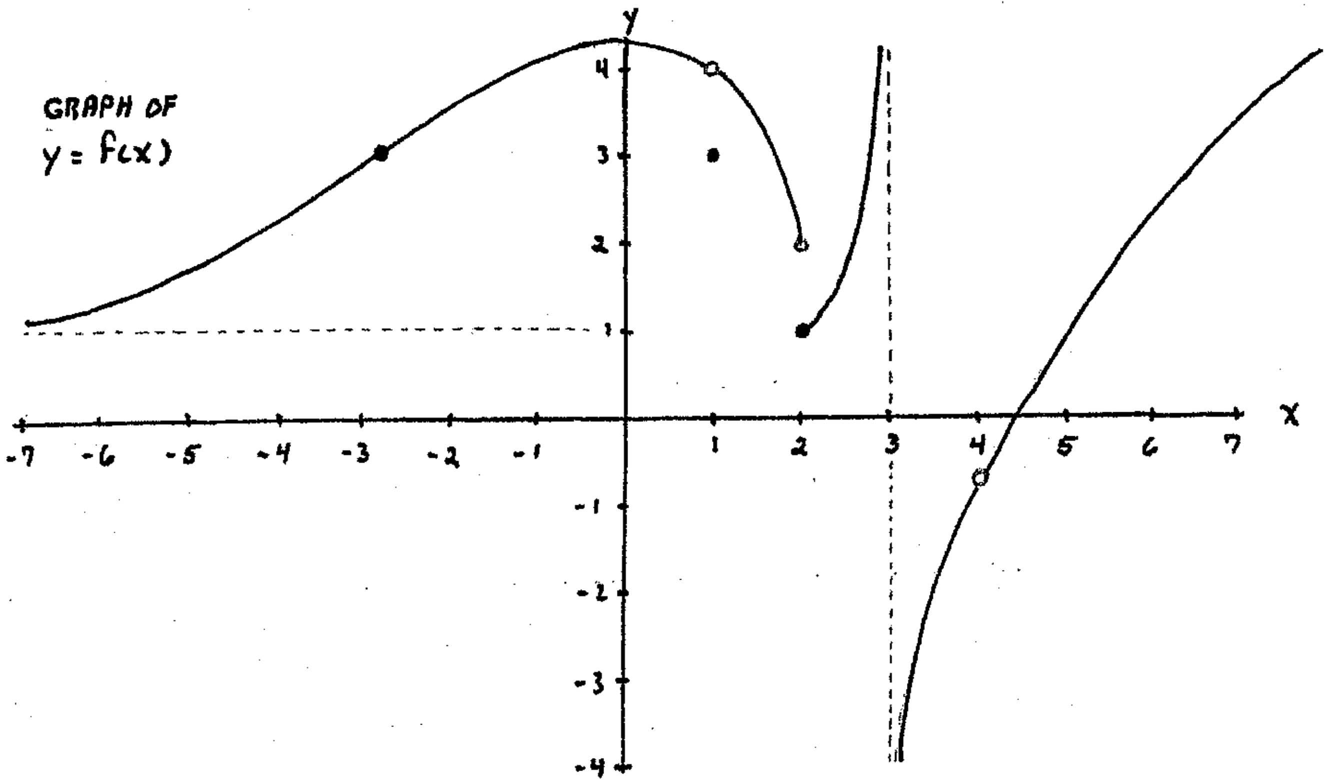


SOME THINGS TO NOTICE :

- $f(1) = 3$
- HOWEVER, WHEN x IS CLOSE TO (BUT NOT EQUAL TO) 1, $f(x)$ IS CLOSE TO 4.
- THE CLOSER x IS TO 1 (BUT STILL NOT EQUAL TO 1), THE CLOSER $f(x)$ IS TO 4.
- $f(x)$ CAN BE MADE AS CLOSE AS YOU LIKE TO 4 JUST BY CHOOSING x CLOSE ENOUGH TO (BUT NOT EQUAL TO) 1.

" THE LIMIT AS x APPROACHES 1 OF $f(x)$ IS 4. "

$$\text{"} \lim_{x \to 1} f(x) = 4 \text{"}$$

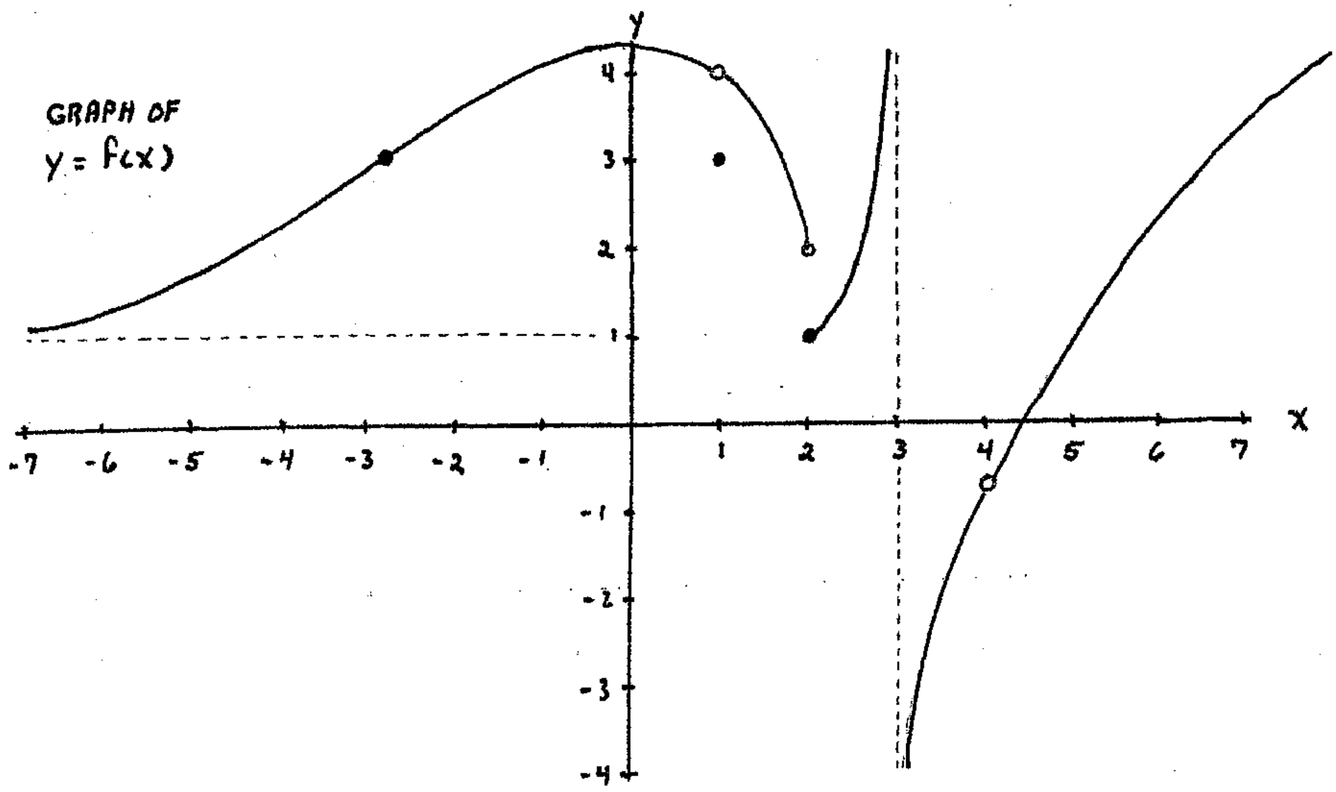


- $f(-3) = 3$
- WHEN x IS CLOSE TO (BUT NOT EQUAL TO) -3, $f(x)$ IS CLOSE TO 3.
- $f(x)$ CAN BE MADE AS CLOSE AS YOU LIKE TO 3 BY CHOOSING x SUFFICIENTLY CLOSE TO (BUT NOT EQUAL TO) -3.

"THE LIMIT AS x APPROACHES -3 OF $f(x)$ IS 3"

"$\lim\limits_{x \to -3} f(x) = 3$"

"$\lim\limits_{x \to -3} f(x) = f(-3)$"



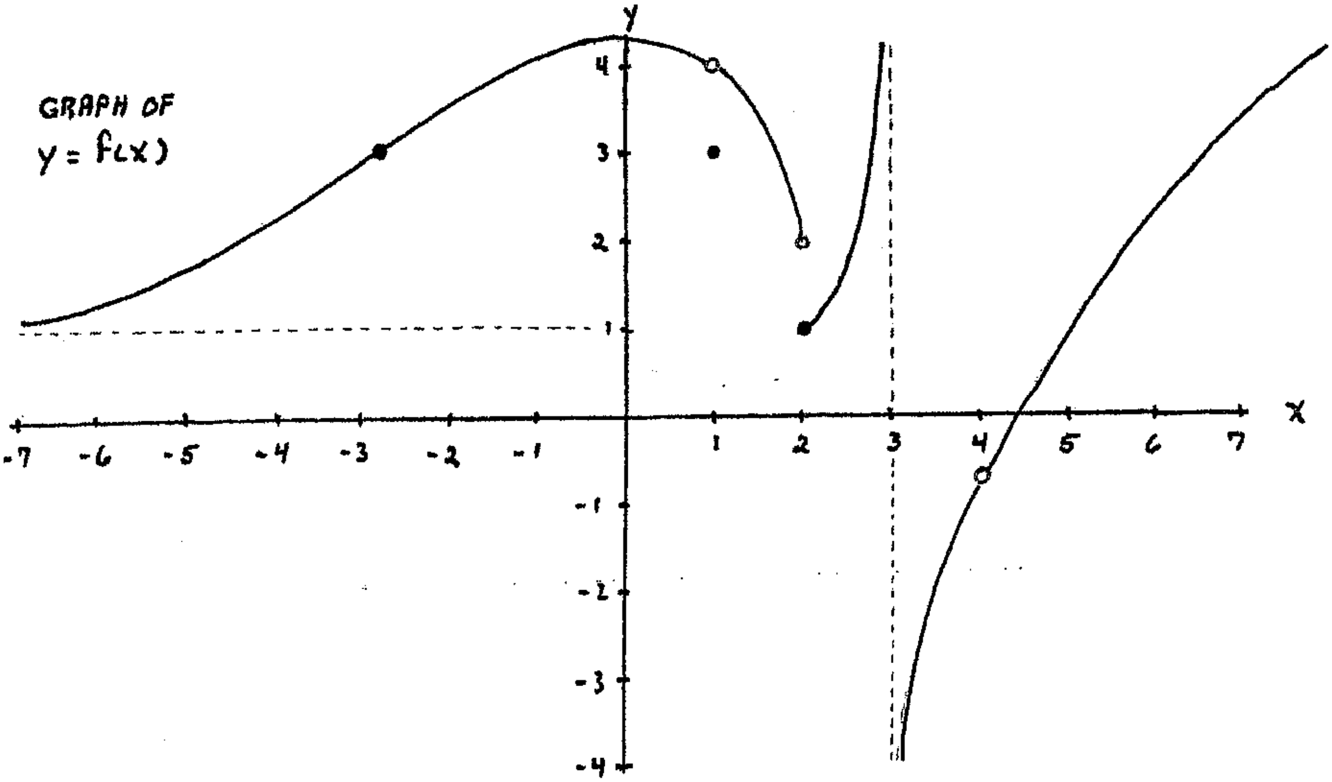
- $f(4)$ IS NOT DEFINED, BUT NEVERTHELESS "$\lim_{x \to 4} f(x) = -1$"

- $f(3)$ IS NOT DEFINED AND THERE IS NO NUMBER THAT $f(x)$ IS CLOSE TO WHEN x IS CLOSE TO 3

  "$\lim_{x \to 3} f(x)$ DOES NOT EXIST"

- $f(2) = 1$, BUT NEVERTHELESS THERE IS NO (SINGLE) NUMBER THAT $f(x)$ IS CLOSE TO WHEN x IS CLOSE TO 2

  "$\lim_{x \to 2} f(x)$ DOES NOT EXIST"



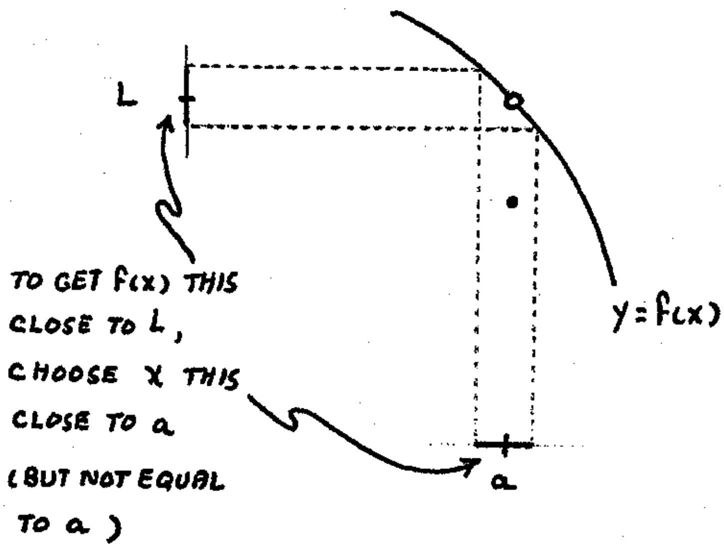
GENERAL DEFINITIONS : LET $f(x)$ BE A FUNCTION AND a A REAL NUMBER (THAT MAY OR MAY NOT BE IN THE DOMAIN OF f). WE SAY THAT THE LIMIT AS x APPROACHES a OF $f(x)$ IS L, WRITTEN

$$\lim_{x \to a} f(x) = L,$$

IF $f(x)$ CAN BE MADE ARBITRARILY CLOSE TO L BY CHOOSING x SUFFICIENTLY CLOSE TO (BUT NOT EQUAL TO) a.

IF NO SUCH NUMBER EXISTS, THEN WE SAY THAT

$$\lim_{x \to a} f(x) \text{ DOES NOT EXIST (DNE).}$$

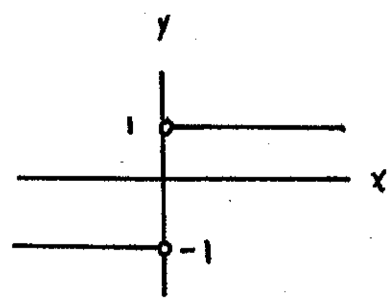


WE WILL START COMPUTING EXAMPLES NEXT TIME, BUT HERE ARE A FEW "OBVIOUS" EXAMPLES:

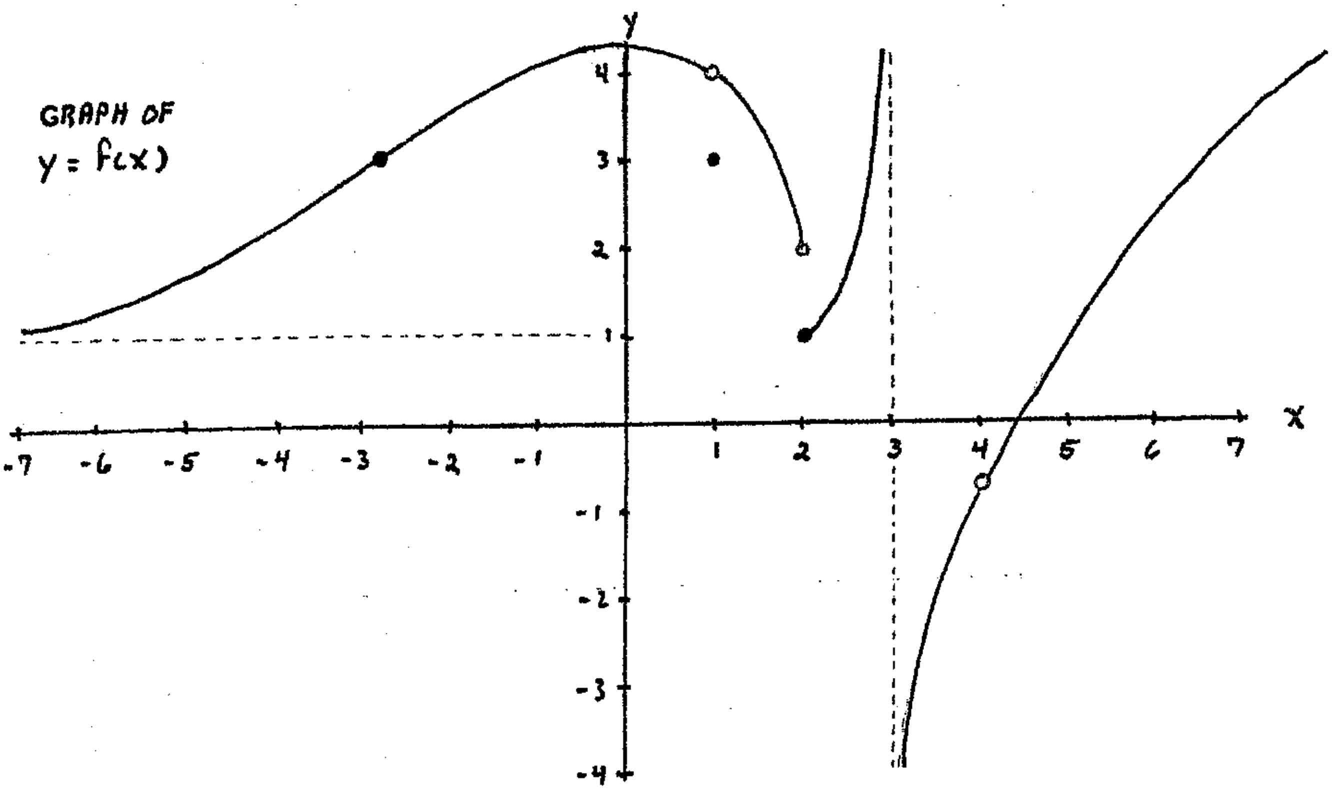
1. $\lim_{x \to 3} 5x = 15$ (5X CAN BE MADE AS CLOSE AS YOU LIKE TO 15 BY CHOOSING X CLOSE ENOUGH TO 3)

2. $\lim_{x \to 0} \frac{1}{x^2}$ DOES NOT EXIST

3. $\lim_{x \to 0} \frac{|x|}{x}$ DOES NOT EXIST



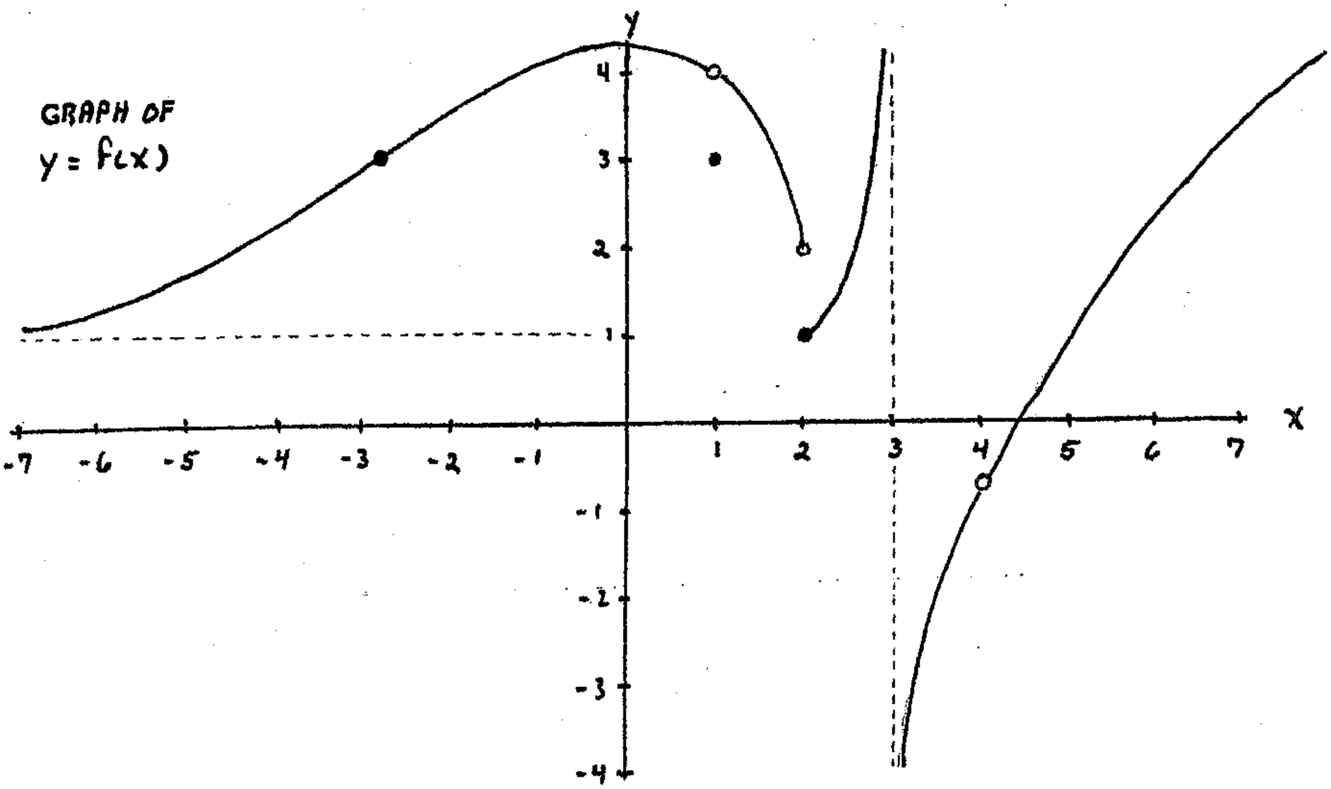
A FEW MORE TYPES OF LIMITS:



ONE-SIDED LIMITS :

FROM BELOW : $\lim_{x \to 2^-} f(x) = 2$ (f(x) CAN BE MADE ARBITRARILY CLOSE TO 2 BY CHOOSING x SUFFICIENTLY CLOSE TO, BUT LESS THAN 2)

FROM ABOVE : $\lim_{x \to 2^+} f(x) = 1$ (f(x) CAN BE MADE ARBITRARILY CLOSE TO 1 BY CHOOSING x SUFFICIENTLY CLOSE TO, BUT GREATER THAN 2)



$\lim_{x \to 3^-} f(x)$ DOES NOT EXIST

$\lim_{x \to 3^+} f(x)$ DOES NOT EXIST

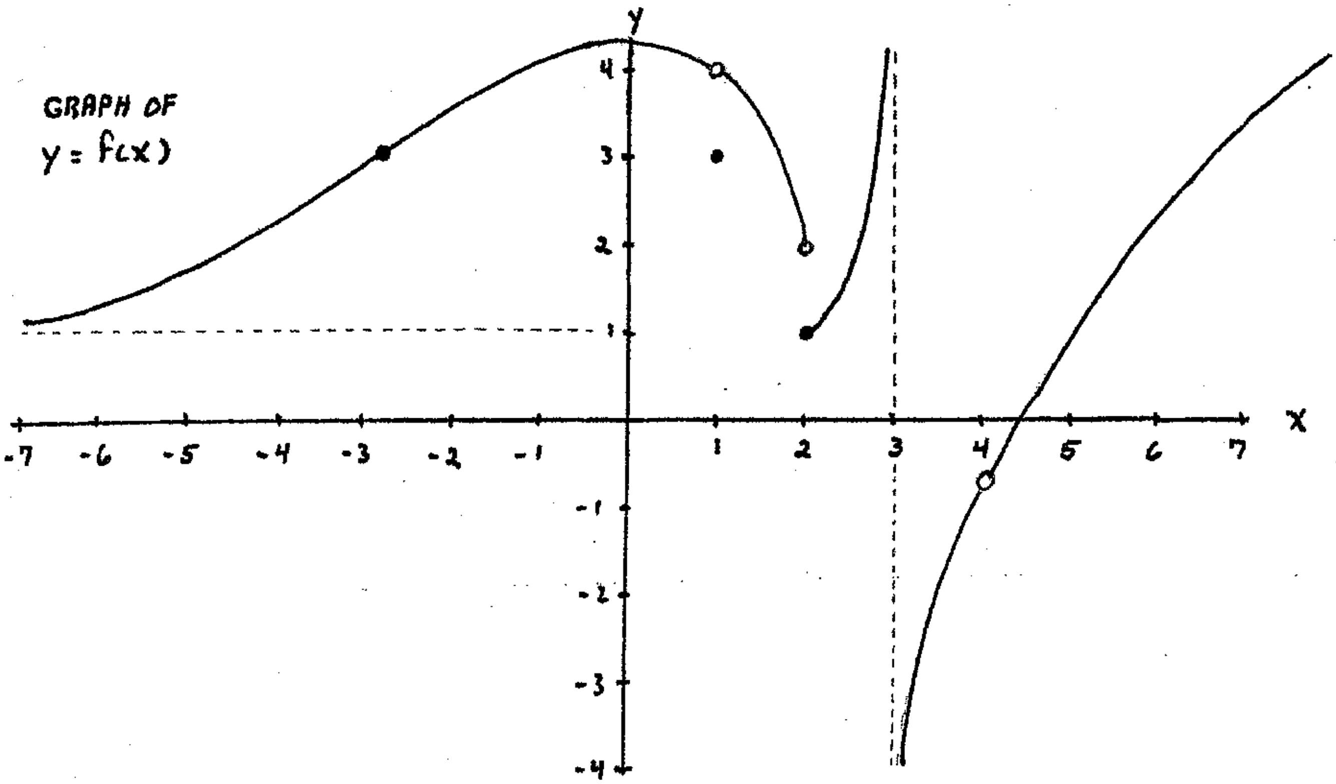
TO DESCRIBE MORE EXPLICITLY THE MANNER IN WHICH THESE LIMITS FAIL TO EXIST WE WILL WRITE

$$\lim_{x \to 3^-} f(x) = \infty$$

AND

$$\lim_{x \to 3^+} f(x) = -\infty$$

NOTE THAT THIS SORT OF THING HAPPENS AT A VERTICAL ASYMPTOTE.



$$\lim_{x \to -\infty} f(x) = 1$$

(f(x) CAN BE MADE ARBITRARILY CLOSE TO 1 BY CHOOSING $|x|$ SUFFICIENTLY LARGE WITH x NEGATIVE)

NOTE THAT THIS INDICATES A <u>HORIZONTAL</u> <u>ASYMPTOTE</u> AT HEIGHT 1.

$\lim_{x \to \infty} f(x)$ DOES NOT EXIST. MORE EXPLICITLY,

$$\lim_{x \to \infty} f(x) = \infty$$