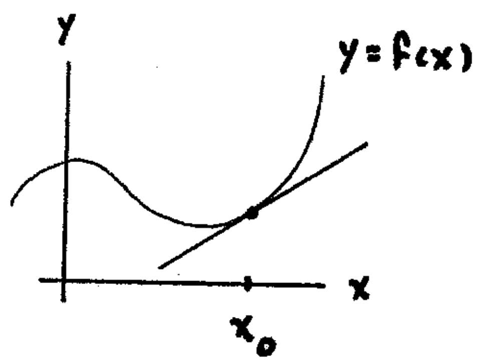
LOCAL LINEAR APPROXIMATIONS



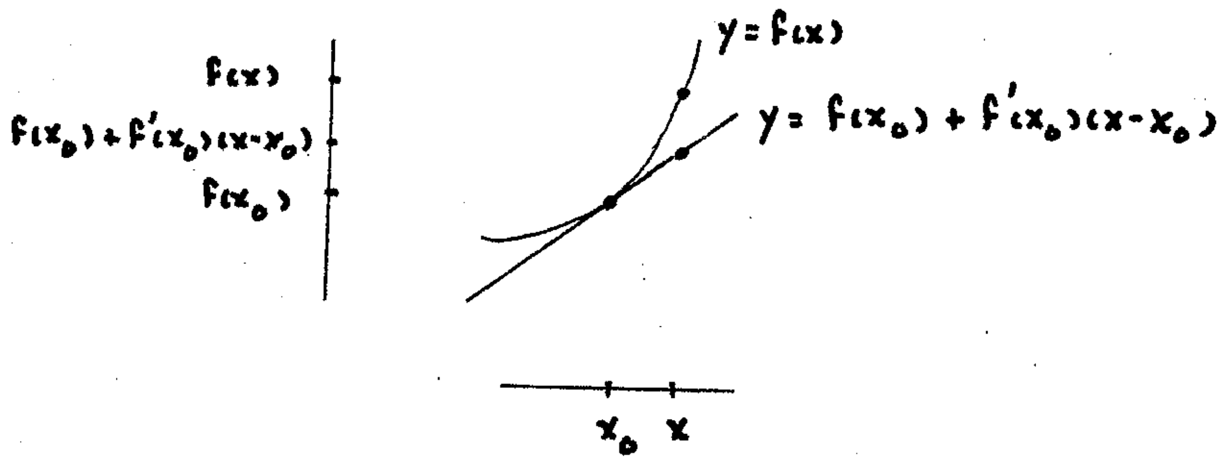
$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

SO, FOR x "CLOSE" TO x_0,

$$f'(x_0) \approx \frac{f(x) - f(x_0)}{x - x_0}$$

WRITE THIS AS

$$\boxed{f(x) \approx f(x_0) + f'(x_0)(x - x_0)}$$



THIS IS CALLED THE LOCAL LINEAR APPROXIMATION OF $f(x)$ NEAR x_0.

ANOTHER WAY OF WRITING THIS (LET $x - x_0 = \Delta x$ SO $x = x_0 + \Delta x$):

$$\boxed{f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x}$$

<u>EXAMPLES</u>:

1. APPROXIMATE THE VALUE OF $\sqrt[3]{1.1}$

LET $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$. WE WANT TO APPROXIMATE $f(1.1)$.

AT $x_0 = 1$ WE KNOW THE VALUES OF f AND f' EXACTLY

($f(x_0) = f(1) = \sqrt[3]{1} = 1$ AND $f'(x_0) = f'(1) = \frac{1}{3\sqrt[3]{1^2}} = \frac{1}{3}$)

SO WE WILL THINK OF 1.1 AS

$$1.1 = 1 + 0.1 = x_0 + \Delta x \qquad (\Delta x = 0.1)$$

THEN

$$\begin{aligned} \sqrt[3]{1.1} &= f(1.1) = f(x_0 + \Delta x) \\ &\approx f(x_0) + f'(x_0)\Delta x = 1 + \tfrac{1}{3}(0.1) \\ &\approx 1.03 \end{aligned}$$

2. APPROXIMATE THE VALUE OF $\sin 2^\circ$

WE HAVE TO WORK IN RADIANS SO CONVERT:

$$360^\circ = 2\pi \text{ rad}$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$2^\circ = \frac{\pi}{90} \text{ rad}$$

THUS, WE'LL APPROXIMATE $\sin \frac{\pi}{90}$.

$$\frac{\pi}{90} = 0 + \frac{\pi}{90} = x_0 + \Delta x$$

WITH $f(x) = \sin x$,

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

$$\sin(x_0 + \Delta x) \approx \sin x_0 + \cos x_0 \, \Delta x$$

$$\sin \frac{\pi}{90} \approx 0 + \cos 0 \left(\frac{\pi}{90}\right)$$

$$\sin \frac{\pi}{90} \approx \frac{\pi}{90} \approx \frac{3.14}{90} \approx 0.01$$

NOTICE THAT THERE'S NOTHING SPECIAL ABOUT $\frac{\pi}{90}$ HERE.
FOR ANY x "NEAR" $x_0 = 0$,

$$\sin x \approx x$$