MATH 235

PROBLEM SOLVING SEMINAR

FALL 2011

DREXEL UNIVERSITY

Instructor: Greg Naber

Office: Korman Center 261

Office Hours: MTWR 3:00-3:50, or by appointment

E-Mail: gln22@drexel.edu

Despite the impression one might get from the undergraduate curriculum, problem solving (in mathematics or anything else) generally has little to do with the mindless regurgitation of facts and formulas. Certainly, one must come prepared with a basic toolkit of known results and techniques, but then it is a matter of one's own ingenuity and, even more, *persistence* that will decide the outcome. This course is designed to provide a modest toolkit appropriate for the sort of problem often encountered in mathematical competitions (such as the Putnam Exam) and an ample opportunity to test your ingenuity and persistence on a wide variety of such problems. We will meet once each week for about two hours. Although the class periods will be devoted almost exclusively to solving problems, each week we will introduce some new topic or technique such as

Induction
Pigeonhole Principle
Modular Arithmetic
Parity
Diophantine Equations
Summation of Series
Generating Functions
Inequalities

or will consider examples of more subtle problems in already familiar areas such as Calculus or Euclidean Geometry. These introductions will be provided to you, but

you will be asked to read them on your own before class so that we can devote our class time to solving problems.

Each week a set of problems will be handed out (which may, or may not be relevant to the topic introduced that week). Your assignment is always to select *one* of these problems and work on it until either you get what you think is a correct solution (in which case you move on to another problem), or a week passes and it is time for the next class. *Never give up!* If you do not manage to solve the problem you should at least come to class bruised and bloodied from the struggle. Most of the time in our weekly meetings will be taken up with your presentation of your own solutions or partial solutions to these problems. Here are a few problems to get you in the mood:

1. **Suppose the plane is colored with two colors; some points are red and some points are blue. Must there be two points an inch apart that have the same color?**

2. **Can an arc of a parabola inside a circle of radius 1 have length greater than 4?**

3. **Show that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive real numbers, then so is $\sum_{n=1}^{\infty} (a_n)^{\frac{n}{n+1}}$.**

4. **Alice and Bob play a game in which they take turns removing some stones from a heap that initially has n stones. The number of stones removed at each turn must be one less than a prime number. The winner is the player who takes the last stone. Alice plays first. Show that there are infinitely many values of n for which Bob has a winning strategy.**

5. **Suppose I is a half-open interval in the real line and f: I → I is a continuous function satisfying the following condition: For each x in I there is a (least) positive integer N(x) such that $f^{N(x)}(x) = x$ [Here we use the notation $f^0(x) = x$, $f^1(x) = f(x)$, $f^2(x) = f(f(x))$, … , $f^n(x) = f(f^{n-1}(x))$, … .] Show that f must be the identity function f(x) = x, but that this is *not* true if I is either open or closed.**

You will be expected to write up and turn in each week the (partial) solution to the problem you worked on that week. Your course grade (which will be one of A, B, or C for all those who see it through to the end) will be determined by the solutions you turn in and your participation during the class periods. The quality of the

exposition is just as important as the quality of the ideas. If your solution is correct, but not neat, clear, detailed and easy to read it will not be considered a complete solution (this is how the Putnam is graded so it is best to get used to it).

The 2011 Putnam Exam will be held on Saturday, December 3, 2011.

Morning Session: 10:00am - 1:00pm

Lunch

Afternoon Session: 3:00pm - 6:00pm

Taking the Putnam is encouraged, but entirely optional.