

PROBLEM SOLVING SEMINAR

PROBLEM SET 1

1. LET  $f$  BE DEFINED ON THE NATURAL NUMBERS BY  $f(1) = 1$  AND, FOR  $n > 1$ ,
- $$f(n) = f(f(n-1)) + f(n - f(n-1)).$$

FIND, WITH PROOF, A SIMPLE EXPLICIT EXPRESSION FOR  $f(n)$ .

2. A DART, THROWN AT RANDOM, HITS A SQUARE TARGET. ASSUMING THAT ANY TWO PARTS OF THE TARGET OF EQUAL AREA ARE EQUALLY LIKELY TO BE HIT, FIND THE PROBABILITY THAT THE POINT HIT IS NEARER THE CENTER THAN TO ANY EDGE.

3. WHICH IS LARGER,  $\int_0^{\pi} \exp(\sin^2 x) dx$ , OR  $\frac{3\pi}{2}$  AND WHY?

4. PROVE THAT THE SUM OF THE INTERIOR ANGLES OF A POLYGON WITH 1001 SIDES IS  $999\pi$ .

5. DOES THERE EXIST AN INFINITE SEQUENCE OF CLOSED DISCS  $D_1, D_2, D_3, \dots$  IN THE PLANE WITH CENTERS  $C_1, C_2, C_3, \dots$ , RESPECTIVELY, SUCH THAT

(i) THE  $C_i$  HAVE NO LIMIT POINT IN THE PLANE,

(ii) THE SUM OF THE AREAS OF THE  $D_i$  IS FINITE, AND

(iii) EVERY LINE IN THE PLANE INTERSECTS AT LEAST ONE OF THE  $D_i$ .