PROBLEM SOLVING SEMINAR

PROBLEM SET 2

1. GIVEN ANY SET OF 10 POSITIVE INTEGERS BETWEEN 1 AND 99 (INCLUSIVE) SHOW THAT THERE ARE TWO DISJOINT NONEMPTY SUBSETS WITH THE SAME SUM.

2. LET $n > 0$ BE AN INTEGER. SHOW THAT FOR ANY POSITIVE REAL NUMBER x,

$$\frac{x^n}{(x+1)^{n+1}} \leq \frac{n^n}{(n+1)^{n+1}}.$$

3. LET $\mathbb{R}^2$ DENOTE THE XY-PLANE AND DEFINE $F : \mathbb{R}^2 \to \mathbb{R}^2$ BY

$$F(x,y) = (4x - 3y + 1,\ 2x - y + 1).$$

DETERMINE $F^{100}(1,0)$, WHERE F^{100} MEANS APPLY F 100 TIMES.

4. GIVEN ANY 5 POINTS ON A SPHERE SHOW THAT 4 OF THEM MUST LIE ON SOME CLOSED HEMISPHERE.

5. WHICH IS LARGER, $99^{50} + 100^{50}$ OR 101^{50}? PROVE THAT YOUR ANSWER IS CORRECT.