

PROBLEM SET 3

1. PROVE THAT $4^{3n+1} + 2^{3n+1} + 1$ IS DIVISIBLE BY 7 FOR ANY NON-NEGATIVE INTEGER n .

2. COMPUTE $\lim_{n \rightarrow \infty} \frac{2 \ln 2 + 3 \ln 3 + \dots + n \ln n}{n^2 \ln n}$

3. DETERMINE WHETHER THE FOLLOWING MATRIX IS SINGULAR OR NONSINGULAR.

$$\begin{pmatrix} 54401 & 57668 & 15982 & 103790 \\ 33223 & 26563 & 23165 & 71489 \\ 36799 & 37189 & 16596 & 46152 \\ 21689 & 55538 & 79922 & 51237 \end{pmatrix}$$

4. A PARTITION OF A SET S IS A COLLECTION OF DISJOINT SUBSETS (PARTS) WHOSE UNION IS S . FOR A PARTITION π OF $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ LET $\pi(x)$ BE THE NUMBER OF ELEMENTS IN THE PART CONTAINING x . PROVE THAT FOR ANY TWO PARTITIONS π AND π' THERE ARE TWO DISTINCT NUMBERS x AND y IN $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ FOR WHICH $\pi(x) = \pi(y)$ AND $\pi'(x) = \pi'(y)$.

5. A LATTICE POINT IN THE PLANE \mathbb{R}^2 IS A POINT WHOSE COORDINATES ARE BOTH RATIONAL. FIND ALL THE LATTICE POINTS ON THE HYPERBOLA

$$x^2 - 2y^2 = 3.$$