

PROBLEM SOLVING SEMINAR

PROBLEM SET 4

1. LET a , b , AND c BE ODD INTEGERS. PROVE THAT THE QUADRATIC POLYNOMIAL $ax^2 + bx + c$ CANNOT HAVE A RATIONAL ROOT.

2. EVALUATE THE SUM $\sum_{i=1}^n i \binom{n}{i}$ FOR ANY INTEGER $n \geq 1$.

3. SHOW THAT, FOR ANY REAL NUMBERS a , b , AND c , THE EQUATION

$$4ax^3 + 3bx^2 + 2cx = a + b + c$$

HAS AT LEAST ONE SOLUTION BETWEEN 0 AND 1.

4. LET A BE A LINEAR TRANSFORMATION ON AN n -DIMENSIONAL VECTOR SPACE V AND SUPPOSE A HAS A SET OF $n+1$ EIGENVECTORS, ANY n OF WHICH ARE LINEARLY INDEPENDENT. PROVE THAT A IS A MULTIPLE OF THE IDENTITY.

5. DEFINE A SEQUENCE BY $a_0 = 1$, TOGETHER WITH THE RULES

$$a_{2n+1} = a_n$$

AND

$$a_{2n+2} = a_n + a_{n+1}$$

FOR EACH $n \geq 0$. PROVE THAT EVERY RATIONAL NUMBER APPEARS IN THE SET

$$\left\{ \frac{a_{n-1}}{a_n} : n \geq 1 \right\} = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{2}, \dots \right\}.$$