PROBLEM SOLVING SEMINAR

PROBLEM SET 6

1. SUM THE SERIES

$$\frac{3}{1\times2\times3} + \frac{5}{2\times3\times4} + \frac{7}{3\times4\times5} + \frac{9}{4\times5\times6} + \cdots$$

2. GIVEN AN INFINITE SET OF POINTS IN THE PLANE PROVE THAT IF ALL OF THE DISTANCES BETWEEN THEM ARE INTEGERS, THEN THE POINTS MUST LIE ON A SINGLE STRAIGHT LINE.

3. CAN A COUNTABLY INFINITE SET HAVE AN UNCOUNTABLE FAMILY OF INFINITE SUBSETS SUCH THAT THE INTERSECTION OF ANY TWO OF THEM IS FINITE. IF NO, PROVE IT; IF SO, FIND AN EXAMPLE.

4. CONSIDER THE POWER SERIES EXPANSION $\frac{1}{1-2x-x^2} = \sum_{n=0}^{\infty} a_n x^n$. PROVE THAT FOR EACH $n \geq 0$ THERE IS AN $m \geq 0$ SUCH THAT

$$a_m = a_n^2 + a_{n+1}^2 .$$

5. LET G BE A GROUP WITH IDENTITY e AND $\phi : G \to G$ A FUNCTION SATISFYING

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

WHENEVER $g_1 g_2 g_3 = e = h_1 h_2 h_3$. PROVE THAT THERE EXISTS AN ELEMENT a IN G SUCH THAT $\psi(x) = a\phi(x)$ IS A HOMOMORPHISM (THAT IS, SATISFIES $\psi(xy) = \psi(x)\psi(y)$ FOR ALL x, y IN G).