PROBLEM SOLVING SEMINAR

PROBLEM SET 7

1. EVALUATE THE SUM $\sum_{j=0}^{n} \sum_{i=j}^{n} \binom{n}{i}\binom{i}{j}$

2. ON A SMALL SQUARE BILLIARD TABLE WITH SIDES OF LENGTH 2 ft A BALL IS PLAYED FROM THE CENTER AND, AFTER REBOUNDING OFF THE SIDES SEVERAL TIMES, GOES INTO A CUP AT ONE OF THE CORNERS. PROVE THAT THE TOTAL DISTANCE TRAVELLED BY THE BALL IS <u>NOT</u> AN INTEGER NUMBER OF FEET.

3. COMPUTE $a_0 + a_1 + \cdots + a_{203}$ IF $a_0 = 2$, $a_1 = 5$ AND, FOR $n \geq 2$, $a_n = 5a_{n-1} - 6a_{n-2}$.

4. PROVE THAT THERE ARE NO PRIMES IN THE SEQUENCE

$$10001,\ 100010001,\ 1000100010001,\ \cdots .$$

5. PROVE THAT THERE IS A <u>UNIQUE</u> FUNCTION f FROM THE SET $\mathbb{R}^+$ OF POSITIVE REAL NUMBERS TO $\mathbb{R}^+$ SUCH THAT

$$f(f(x)) = 6x - f(x)$$