

PROBLEM SOLVING SEMINAR

PROBLEM SET 9

1. FIND THE LEAST POSSIBLE AREA OF A CONVEX SET IN THE PLANE THAT INTERSECTS BOTH BRANCHES OF THE HYPERBOLA $xy=1$ AND BOTH BRANCHES OF THE HYPERBOLA $xy=-1$ (A SET S IN THE PLANE IS CONVEX IF FOR ANY TWO POINTS IN S THE LINE SEGMENT CONNECTING THEM IS CONTAINED IN S).
2. LET k BE A POSITIVE INTEGER. THE n^{TH} DERIVATIVE OF $\frac{1}{x^{k-1}}$ HAS THE FORM $\frac{P_n(x)}{(x^{k-1})^{n+1}}$, WHERE $P_n(x)$ IS A POLYNOMIAL. FIND $P_n(1)$.
3. LET $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ BE A SURJECTIVE MAP WITH THE PROPERTY THAT IF THE POINTS A, B, C ARE COLLINEAR, THEN $f(A), f(B), f(C)$ ARE COLLINEAR. PROVE THAT f IS BIJECTIVE.
4. LET N_n DENOTE THE NUMBER OF ORDERED n -TUPLES OF POSITIVE INTEGERS (a_1, a_2, \dots, a_n) SUCH THAT
$$\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} = 1.$$
 DETERMINE WHETHER N_{10} IS EVEN OR ODD.
5. CONSIDER A PAPER PUNCH THAT CAN BE CENTERED AT ANY POINT OF THE PLANE AND THAT, WHEN OPERATED, REMOVES FROM THE PLANE PRECISELY THOSE POINTS WHOSE DISTANCE FROM THE CENTER IS IRRATIONAL. HOW MANY PUNCHES ARE NEEDED TO REMOVE EVERY POINT?