1. **Suppose the plane is colored with two colors; some points are red and some points are blue. Must there be two points an inch apart that have the same color?**

2. **Can an arc of a parabola inside a circle of radius 1 have length greater than 4?**

3. **Show that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive real numbers, then so is $\sum_{n=1}^{\infty} (a_n)^{\frac{n}{n+1}}$.**

4. **Alice and Bob play a game in which they take turns removing some stones from a heap that initially has n stones. The number of stones removed at each turn must be one less than a prime number. The winner is the player who takes the last stone. Show that there are infinitely many values of n for which Bob has a winning strategy.**

5. **Suppose I is a half-open interval in the real line and f: I → I is a continuous function satisfying the following condition: For each x in I there is a (least) positive integer N(x) such that $f^{N(x)}(x) = x$ [Here we use the notation $f^0(x) = x$, $f^1(x) = f(x)$, $f^2(x) = f(f(x))$, … , $f^n(x) = f(f^{n-1}(x))$, … .] Show that f must be the identity function f(x) = x, but that this is *not* true if I is either open or closed.**