The goal here is to provide an introduction to the physical and mathematical foundations of quantum mechanics. It is addressed to those who have been trained in modern mathematics and whose background in physics may not extend much beyond F = mA, but for whom the following sorts of questions require more than a perfunctory response. What are the physical phenomena that forced physicists into such a radical reevaluation of the very beautiful and quite successful ideas bequeathed to them by the founding fathers of classical physics? Why did this reevaluation culminate in a view of the world that is probabilistic and formulated in terms of Hilbert spaces and self-adjoint operators? Where did the Planck constant come from? What are the basic assumptions of quantum mechanics? Are they consistent? What motivated them? What objections might be raised to them? Where did the Heisenberg algebra come from? What motivated Feynman to introduce his path integral? Is it really an "integral"? Does it admit a rigorous mathematical definition? Why does one distinguish two "types" of particles in quantum mechanics (bosons and fermions)? Why and how are they treated differently? In what sense does supersymmetry provide a more unified picture of the two types? One need not know the answers to all of these questions in order to study the mathematical formalism of quantum mechanics, but for those who would like to know we will try to provide some answers or, at least, some food for thought. As to the mathematical formalism itself, we will provide careful, detailed and rigorous treatments of just a few of the simplest and most fundamental systems with which quantum mechanics deals in the hope that this will lay the foundation for a deeper study of the physical applications to be found in the literature.

Rather detailed discussions of Lagrangian and Hamiltonian mechanics, Koopman's formulation of classical statistical mechanics, electromagnetic radiation, blackbody radiation, the photoelectric effect, 2-slit experiments, Heisenberg's 1925 paper, Brownian motion, the Stern-Gerlach experiment and spin one-half should, we hope, satisfy the needs of those for whom the physical background of quantum mechanics is of more than historical interest. Although a general familiarity with functional analysis is assumed, rather thorough synopses of such topics as unbounded, self-adjoint operators and spectral theory, tempered distributions and Sobolev spaces, and strongly continuous semigroups of operators are included and illustrated with numerous and quite detailed examples relevant to quantum theory. A concerted effort has been made to provide ample references for the background material that has not been treated in detail. The quantum mechanical applications revolve around the free particle and the harmonic oscillator. For each of these we study both the canonical quantization and the Feynman path integral. Also included are discussions of various rigorous self-adjointness theorems for quantum Hamiltonians, rigorous versions of the Ehrenfest Theorem, the Groenewold-Van Hove Theorem, the Feynman-Souriau Formula, and the relation between the Feynman path integral and the Wiener measure on path space. The fermionic and supersymmetric variants of the harmonic oscillator are also discussed in detail. Abstracting the essential features of the supersymmetric oscillator leads to the general notion of an N=2 supersymmetry and Witten's Hodge theory example is carefully described. Appendices on Gaussian Integrals, the Morse Lemma, and Stationary Phase Approximation are also included.