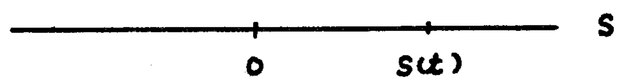
RECTILINEAR MOTION

CONSIDER AN OBJECT MOVING ALONG A STRAIGHT LINE (CAR MOVING ALONG A STRAIGHT HIGHWAY, ROCK FALLING VERTICALLY, MASS ATTACHED TO A VIBRATING SPRING, ETC.)

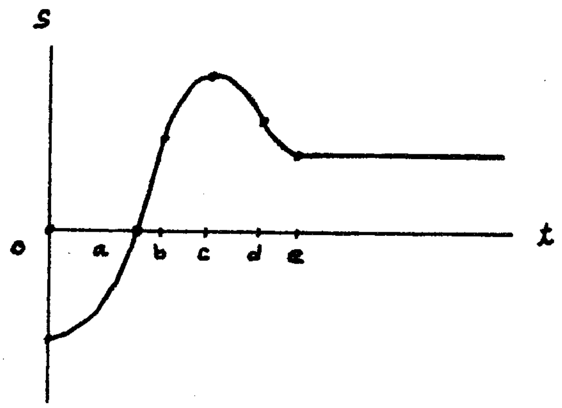
CALL THE STRAIGHT LINE THE "S-AXIS" AND LET THE OBJECT'S S-COORDINATE AT TIME t BE DENOTED

$$s(t)$$



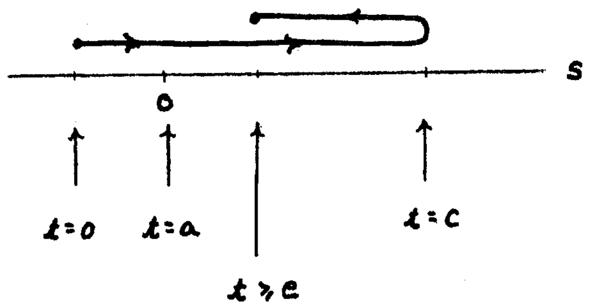
THE GRAPH OF THE FUNCTION $s(t)$ IN THE ts-PLANE IS THE <u>POSITION VERSUS TIME CURVE</u> OF THE OBJECT.

E.G.,



INTERPRETATION :

$t = 0$ TO $t = a$	OBJECT IS TO THE LEFT OF THE ORIGIN AND MOVING TOWARD THE ORIGIN.
$t = a$ TO $t = c$	TO RIGHT OF THE ORIGIN, MOVING AWAY FROM IT.
$t = c$ TO $t = e$	TURNS AROUND AND HEADS BACK TOWARD THE ORIGIN.
$t = e$	COMES TO A STOP BEFORE ARRIVING AT THE ORIGIN AND STAYS THERE.



<u>INSTANTANEOUS</u> <u>VELOCITY</u> : $v(t) = S'(t)$ = SLOPE OF TANGENT TO $S(t)$

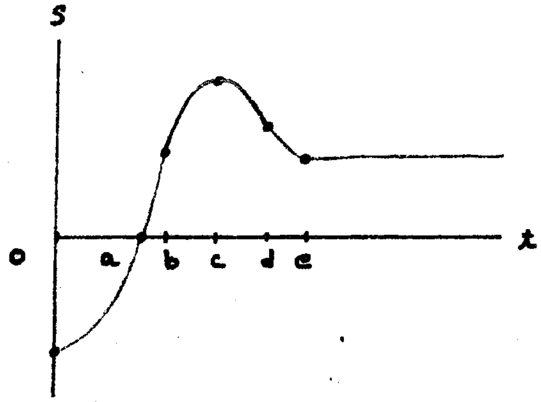
$v(t) > 0$: MOVING TO THE RIGHT (POSITION CURVE INCREASING)

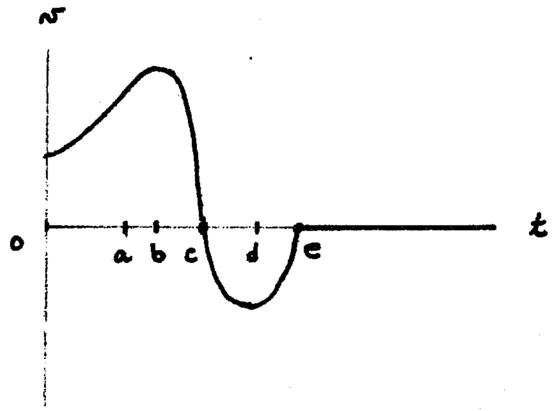
$v(t) < 0$: MOVING TO THE LEFT (POSITION CURVE DECREASING)

INSTANTANEOUS SPEED : $|v(t)|$

THE GRAPH OF THE FUNCTION $v(t)$ IN THE tv-PLANE IS THE VELOCITY VERSUS TIME CURVE OF THE OBJECT.

E.G.,





INTERPRETATION :

$t = 0$ TO $t = b$ VELOCITY STARTS OUT POSITIVE AND BECOMES LARGER AND LARGER UNTIL IT REACHES A MAXIMUM VALUE AT $t = b$.

$t = b$ TO $t = c$ VELOCITY DECREASES FROM ITS MAXIMUM POSITIVE VALUE AT $t = b$ TO ZERO AT $t = c$

$t = c$ to $t = d$ — VELOCITY STARTS OUT AT ZERO AND BECOMES INCREASINGLY NEGATIVE UNTIL IT REACHES ITS MOST NEGATIVE VALUE AT $t = d$

$t = d$ to $t = e$ — VELOCITY BECOMES LESS AND LESS NEGATIVE UNTIL IT REACHES THE VALUE ZERO AT $t = e$

$t \geq e$ — VELOCITY IS ZERO

THUS, STARTING FROM ITS POSITION TO THE LEFT OF THE ORIGIN AT $t = 0$ IT HEADS TOWARD THE ORIGIN AT INCREASING SPEED, ZOOMS PAST THE ORIGIN AT $t = a$, CONTINUES TO PICK UP SPEED UNTIL $t = b$ WHEN IT STARTS TO SLOW DOWN, COMING TO AN INSTANTANEOUS STOP AT $t = c$ SO THAT IT CAN TURN AROUND AND HEAD BACK TOWARD THE ORIGIN, INCREASING ITS SPEED UNTIL $t = d$ WHEN IT STARTS TO SLOW DOWN, COMING TO A COMPLETE STOP AT $t = e$ BEFORE ARRIVING AT THE ORIGIN.

<u>INSTANTANEOUS</u> <u>ACCELERATION</u> : $a(t) = v'(t) = s''(t)$

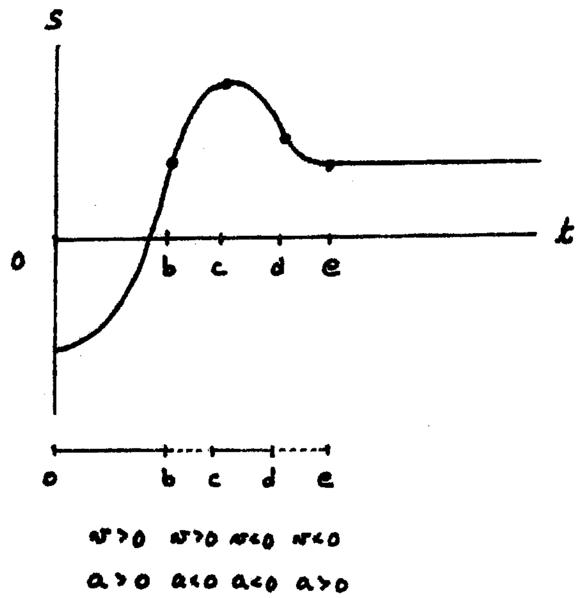
$a(t) > 0$: VELOCITY INCREASING (GRAPH OF $s(t)$ IS CONCAVE UP)

$a(t) < 0$: VELOCITY DECREASING (GRAPH OF $s(t)$ IS CONCAVE DOWN)

NOTE THAT

$a(t)$ AND $v(t)$ HAVE THE SAME SIGN $\Rightarrow$ SPEEDING UP

$a(t)$ AND $v(t)$ HAVE OPPOSITE SIGNS $\Rightarrow$ SLOWING DOWN



<u>EXAMPLES</u> :

1. ANALYZE THE MOTION OF AN OBJECT WHOSE POSITION FUNCTION IS

$$s(t) = 2t^3 - 21t^2 + 60t + 3$$

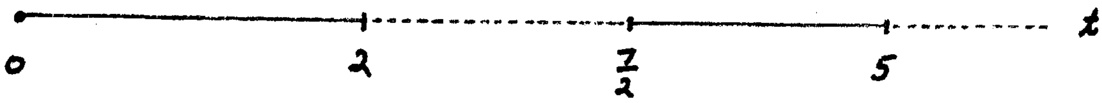
FOR $t \geq 0$.

$$v(t) = s'(t) = 6t^2 - 42t + 60 = 6(t^2 - 7t + 10) = 6(t-2)(t-5)$$

$$= 0 \text{ WHEN } t = 2, 5$$

$$a(t) = s''(t) = 12t - 42 = 6(2t - 7)$$

$$= 0 \text{ WHEN } t = \frac{7}{2}$$

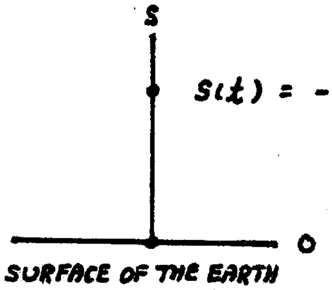


$v > 0$	$v < 0$	$v < 0$	$v > 0$
$a < 0$	$a < 0$	$a > 0$	$a > 0$

$v = 0$ $\qquad\qquad$ $v = 0$

RIGHT $\longrightarrow$	LEFT $\longleftarrow$	LEFT $\longleftarrow$	RIGHT $\longrightarrow$
SLOWING DOWN	SPEEDING UP	SLOWING DOWN	SPEEDING UP

2.



$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$ (WE WILL PROVE THIS SHORTLY)

WHERE s_0 IS THE HEIGHT AT $t = 0$, v_0 IS THE VELOCITY AT $t = 0$ AND g IS A CONSTANT (32 FT/SEC^2 OR 9.8 M/SEC^2)

$$v(t) = s'(t) = -gt + v_0$$

$$a(t) = v'(t) = -g \quad \text{(CONSTANT ACCELERATION)}$$

(a) A ROCK, DROPPED FROM AN UNKNOWN HEIGHT, STRIKES THE GROUND WITH A SPEED OF 24 M/SEC. FIND THIS UNKNOWN INITIAL HEIGHT.

SOLUTION TO (a) : "DROPPED" MEANS $v_0 = 0$ SO

$$s(t) = -\frac{1}{2}gt^2 + s_0$$

$$v(t) = -gt$$

WE'RE SUPPOSED TO FIND S_0.

"STRIKES THE GROUND" MEANS $S = 0$. THIS HAPPENS WHEN

$$-\frac{1}{2}gt^2 + S_0 = 0$$

$$t = \sqrt{\frac{2S_0}{g}}$$

SO, AT THIS TIME, $v = -24\ m/sec.$

$$-24 = -g\sqrt{\frac{2S_0}{g}}$$

$$24 = \sqrt{2gS_0}$$

$$S_0 = \frac{24^2}{2g} = \frac{576}{2(9.8)} = 29.39\ m$$

(b) A BALL IS THROWN UPWARD FROM A HEIGHT S_0 WITH AN INITIAL VELOCITY OF v_0. SHOW THAT THE MAXIMUM HEIGHT OF THE BALL IS

$$S_{max} = S_0 + \frac{v_0^2}{2g}$$

SOLUTION TO (b) : THE MAXIMUM VALUE OF $s(t)$ OCCURS AT A POINT WHERE THE VELOCITY IS ZERO

$$s'(t) = -gt + v_0 = 0 \Rightarrow$$

$$t = \frac{v_0}{g}$$

AT THIS TIME, THE HEIGHT IS

$$s\left(\frac{v_0}{g}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) + s_0$$

$$= s_0 - \frac{v_0^2}{2g} + \frac{v_0^2}{g}$$

$$= s_0 - \frac{v_0^2}{2g} + \frac{2v_0^2}{2g}$$

$$= s_0 + \frac{v_0^2}{2g}$$

<u>A QUESTION FOR YOU</u>: NOLAN RYAN COULD THROW A BASEBALL 102 MI/HR. IF HE THREW THE BALL STRAIGHT UP, RELEASING IT FROM A HEIGHT OF 7 FT, COULD HE HIT THE CEILING OF THE ASTRODOME (208 FT HIGH)?

VELOCITY IS THE DERIVATIVE OF POSITION, I.E.,

$s(t)$ IS AN ANTIDERIVATIVE FOR $v(t)$

ACCELERATION IS THE DERIVATIVE OF VELOCITY, I.E.,

$v(t)$ IS AN ANTIDERIVATIVE FOR $a(t)$.

<u>EXAMPLE</u> (<u>UNIFORMLY ACCELERATED MOTION</u>): SUPPOSE

$$\boxed{a(t) = a \quad \text{(A CONSTANT)}}$$

$$v'(t) = a$$

$v(t)$ IS AN ANTIDERIVATIVE FOR THE CONSTANT FUNCTION a

$$\int a\,dt = at + C$$

$v(t) = at + C$ FOR SOME CONSTANT C

$v(0) = a\cdot 0 + C \Rightarrow C = v(0)$ SO

$v(t) = at + v(0)$

<u>NOTE</u> : CUSTOMARY TO WRITE $v(0) = v_0$

$$\boxed{v(t) = at + v_0}$$

BUT NOW, $s'(t) = at + v_0$

$s(t)$ IS AN ANTIDERIVATIVE FOR $at + v_0$

$$\int (at + v_0)\,dt = \tfrac{1}{2}at^2 + v_0 t + C$$

$s(t) = \frac{1}{2}at^2 + v_0 t + C$ FOR SOME CONSTANT C

$s(0) = \frac{1}{2}a\cdot 0^2 + v_0\cdot 0 + C \Rightarrow C = s(0) = s_0$ SO

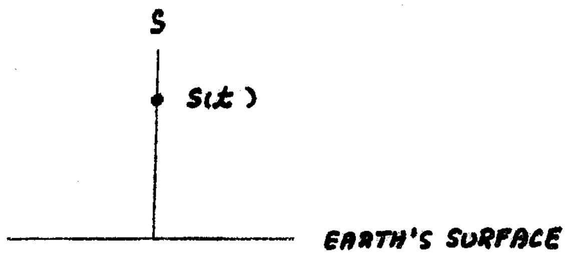
$$\boxed{s(t) = \tfrac{1}{2}at^2 + v_0 t + s_0}$$

a = CONSTANT ACCELERATION

v_0 = INITIAL VELOCITY

s_0 = INITIAL POSITION

E.G., FOR FREE FALL



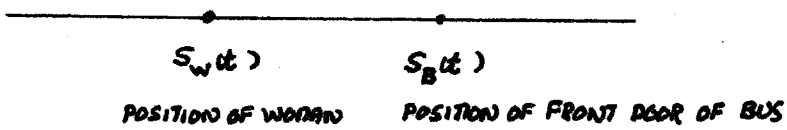
$$a = -g$$

GIVES THE FORMULAS WE USED EARLIER:

$$s(t) = -\frac{1}{2} g t^2 + v_0 t + s_0$$

$$v(t) = -gt + v_0$$

ANOTHER EXAMPLE: A BUS IS STOPPED TO PICK UP PASSENGERS. A WOMAN IS RUNNING AT 5 m/sec TO CATCH IT. WHEN SHE IS 13 m BEHIND THE FRONT DOOR THE BUS PULLS AWAY WITH A CONSTANT ACCELERATION OF 1 m/sec^2. WILL SHE CATCH THE BUS?



SHE WILL CATCH THE BUS IF, FOR SOME t,

$$s_W(t) = s_B(t)$$

ASSUME: $s_W(0) = 0$ $\quad$ $s_B(0) = 13$ $\quad$ $v_B(0) = 0$

$$v_W(t) = 5$$

$$s_W(t) = 5t + s_W(0) = 5t$$

$$a_B(t) = 1$$

$$v_B(t) = 1 \cdot t + v_B(0) = t$$

$$s_B(t) = \frac{1}{2} t^2 + s_B(0) = \frac{1}{2} t^2 + 13$$

$$S_W(t) = S_B(t)$$
$$5t = \tfrac{1}{2}t^2 + 13$$
$$t^2 - 10t + 26 = 0$$

BUT THIS HAS NO REAL SOLUTIONS SINCE $b^2 - 4ac = (-10)^2 - 4(1)(26) = -4$.

SHE WON'T CATCH THE BUS.

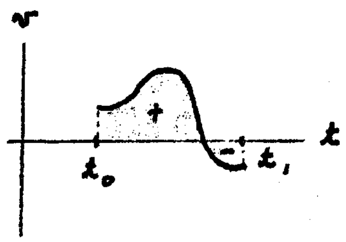
ANOTHER EXAMPLE:

$s(t)$ = POSITION

$v(t) = s'(t)$ = VELOCITY

$\int_{t_0}^{t_1} v(t)\,dt = s(t_1) - s(t_0)$ = DISPLACEMENT (OR CHANGE IN POSITION) BETWEEN TIMES t_0 AND t_1

= NET SIGNED AREA OF THE VELOCITY CURVE OVER $[t_0, t_1]$



QUESTION: AT TIME t_1 IS THE OBJECT FARTHER TO THE RIGHT OR FARTHER TO THE LEFT THAN IT WAS AT TIME t_0?

$\int_{t_0}^{t_1} |v(t)|\,dt$ = TOTAL DISTANCE TRAVELED (IN EITHER DIRECTION) BETWEEN TIME t_0 AND TIME t_1